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Increased Power Density, Efficiency, and Durability With MEGAGEARS® and UNIMEGAGEARS®

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ABSTRACT

The vast majority of gear teeth manufactured around the world have an involute profile developed by the Austrian scientist Leonhard Euler (1707-1783). The fundamental feature of the involute profile principle is that it provides constant angular velocity. This is an absolute necessity because in absence of constant rotational motion, vibrations and/or high stress can be generated that can lead to damage and even destruction of the gears.

Power Engineering and Manufacturing, Ltd. developed advanced design gear teeth that have a higher power density, durability, and efficiency that are registered under the MEGAGEARS® and UNIMEGAGEARS® trade names.

INTRODUCTION

Gear teeth have a curvature that follows the involute profile. The involute can be defined as a curve whose radius of curvature increases as it moves further away from the base circle diameter. (See Figure 1)

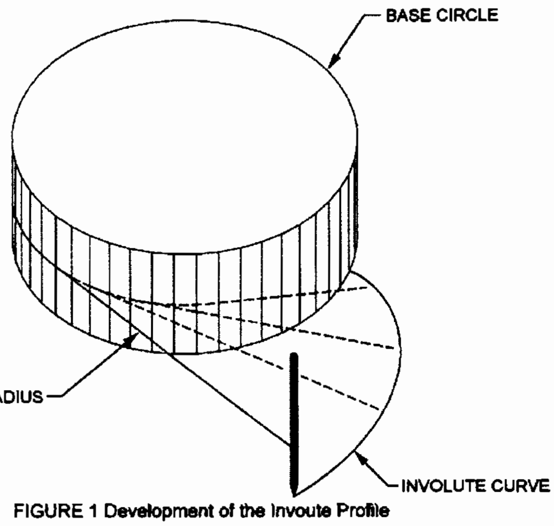


FIGURE 1 Development of the Invoute Profile

The advantage of the involute curvature is that it provides constant angular velocity. This is very important. The gear teeth can use any portion of the involute profile to provide proper load transfer from the tooth on the first gear to the mating tooth on the second gear. (See Figure 2)

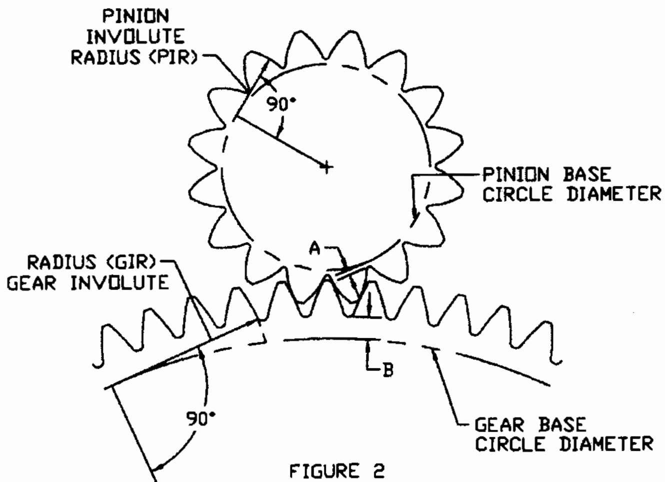


FIGURE 2
Gears In Mesh

Note in Figure 2, the distance (A) from the pinion base circle to the root is smaller than distance (B) from the gear base circle to the root.

The larger the number of teeth, the larger the involute radius of curvature. This is very important for surface fatigue properties of the gear teeth. Often it is desirable to make the pinion with a much smaller number of teeth than the gear to obtain a large reduction ratio. The pinion in a set of gears with a reduction ratio greater than 3:1 or 4:1 has two detrimental properties:

1) The pinion accumulates more fatigue load cycles than the gear.

2) The pinion tooth is positioned on the involute radius of curvature closer to the base circle than the gear tooth (PIR<GIR) which can substantially raise the surface

compressive stress at the start of active profile (SAP). See Figure 2.

The MEGAGEAR® and the UNIMEGAGEAR® design method is to position the pinion tooth as far away as possible from the base circle and to maximize the pinion involute radius at the start of active profile, without making it larger than the gear involute radius at the start of active profile, this will maximize the surface fatigue life. See Figure 3.

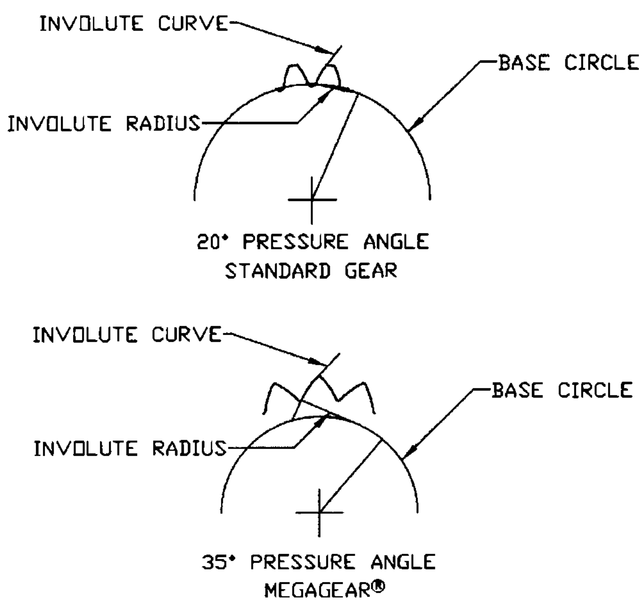


FIGURE 3
Comparison of a Standard Gear and a MEGAGEAR®

It can be seen in Figure 3, that moving the position of the tooth further away from the base circle makes the involute radius at the start of active profile on the MEGAGEAR® larger than the involute radius at the start of active profile of the standard gear. It will be shown later how this increases the surface fatigue life.

THE INVOLUTE

The involute curve can best be understood if we imagine that a string wound on a drum (that is in reality the gear base circle) has a pencil attached to one end and is scribing a line as it is being unwound off the drum and holding the string tight. (See Figure 1). It is important to note that at any point of contact when the unwound section of the string that holds the pencil is held tight, the string forms a tangent with the base circle. This is a fundamental principle in the development of the involute profile curve. The involute's line can be defined as a "curved line" whose radius of curvature increases the further away it is from the base circle. The unwound string illustrates that the involute profile can exist only outside the base circle. The involute profile cannot exist inside the base circle because a tight string cannot be unwound inside the base circle. For this reason, it is desirable to always make the root diameter larger than the base circle diameter. Whenever the root diameter is smaller than the base circle diameter, the tooth will have undercut. Undercut is undesirable, it is not desirable to have because it weakens the tooth in bending and may reduce the profile contact ratio.

It is not recommended to use the involute profile all the way to the base circle because the closer the contact point is to the base circle, the smaller the involute radius becomes until it is zero at the base circle. The gear teeth, when in mesh, will use a different section of the involute curve and as long as they use the involute profile, they will always operate at a constant angular velocity. Good gear teeth may be constructed by using various sections of the involute profile. The further away the contact is from the base circle, the more power it can carry with a thicker oil film.

The practice of designing gears with a profile contact ratio of two or greater is useful only in reducing noise. Such teeth are weaker in bending, have a higher surface compressive stress and are capable of carrying a lower torque with a lower efficiency. MEGAGEARS® and UNIMEGAGEARS® have a near one profile contact ratio, can carry a substantially higher torque, and have a much lower bending stress that nearly eliminates the possibility of breaking an entire tooth off at the root. Because the MEGAGEARS® and UNIMEGAGEARS® have a higher spring rate, they have a lower deflection under load which results in quieter operation.

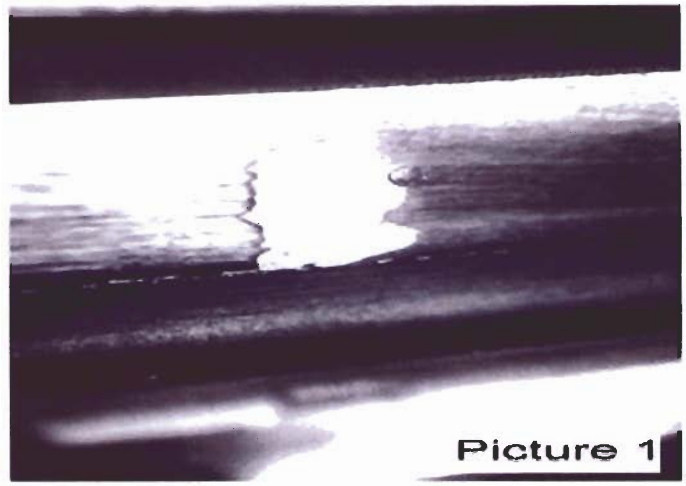
Care must be exercised to avoid making a weak addendum because the further away the tooth is from the base circle, the more pointed the tooth becomes. When the practice of pinion long addendum and gear short addendum was introduced to reduce bending stress (due to undercut) by moving the pinion tooth radially out until there is no more undercut, accidentally and probably unknowingly the surface fatigue durability was also increased because the point of pinion start of active profile was moved further above the base circle. When the start of active profile is at the base circle, the involute radius at that point is zero, the surface compressive stress is infinite, and spalling starts immediately. This can be proven by substituting "r1" with "zero" in Equation 1.

Equation 1: Surface Compressive Stress =

$$SCS = \sqrt{\frac{\mathrm{E}}{\pi \cdot \left(1+\nu^2\right)} \cdot \frac{T}{BR} \cdot \frac{LLOA}{FW \cdot PCR \cdot r1 \cdot r2 \cdot 2}}$$

(Terms are defined at the end of the paper)

AGMA surface ratings are based on calculations of the surface compressive stress and fatigue at the pitch diameter. If we examine Pictures Number 1 and 2, it can be seen that slightly above the tooth mid-point where the generating pitch diameter is expected to be, there is no pitting or wear except for one large pit. The large pit is an outgrowth from the line of small pits at the start of active profile in a manner similar to how pot holes grow on the road as more and more tires continue to crumble the pit edges. This proves that this failure could not have been predicted by the AGMA standards, because these standards do not calculate the surface fatigue life at the start of active profile. The gear life could have been increased substantially by modifying the gear teeth of both the pinion and the gear to reduce the surface compressive stress, as is the practice at Power Engineering and Manufacturing Ltd.


Picture 1

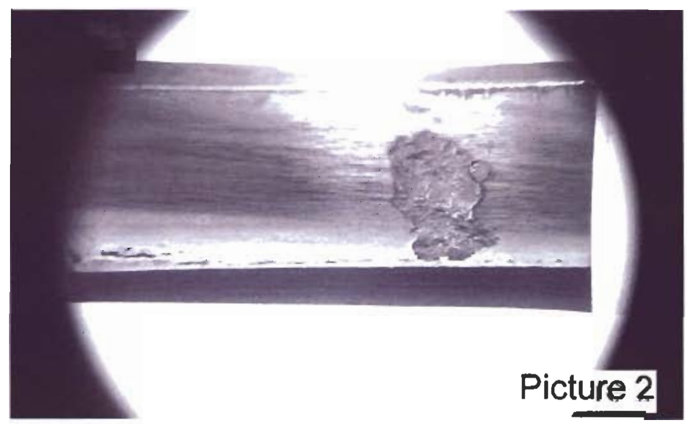

Picture 2

TABLE 1 Comparison of Surface Compressive Stresses in Standard 25° PA Gears and 33.7° PA MEGAGEARS®

	Working SCS at Pitch Diameter		SCS at Point of Single Tooth Contact		SCS at Start of Active Profile	
Gear Pressure Angle(Degrees)	25.0	33.7	25.0	33.7	25.0	33.7
Pinion Surface Compressive Stress(PSI)	129,000	118,000	143,000	123,000	172,000	125,000
Pinion Surface Fatigue Life(Percent of 25° PA at Working Pitch Diameter)	**100%**	**181%**	**50%**	**137%**	**15%**	**123%**

AGMA SURFACE DURABILITY RATING

Figure 4 shows gears designed in accordance with AGMA guidelines.

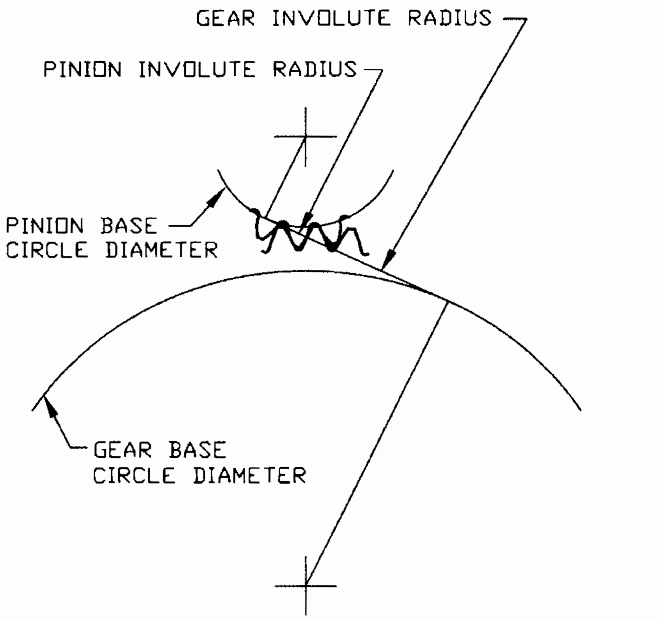


FIGURE 4 AGMA 25° PRESSURE ANGLE GEARS

Figure 5 shows a gear set designed in accordance with MEGAGEAR® guidelines. Both sets of gears have the same parameters to make them equivalent for a valid comparison.

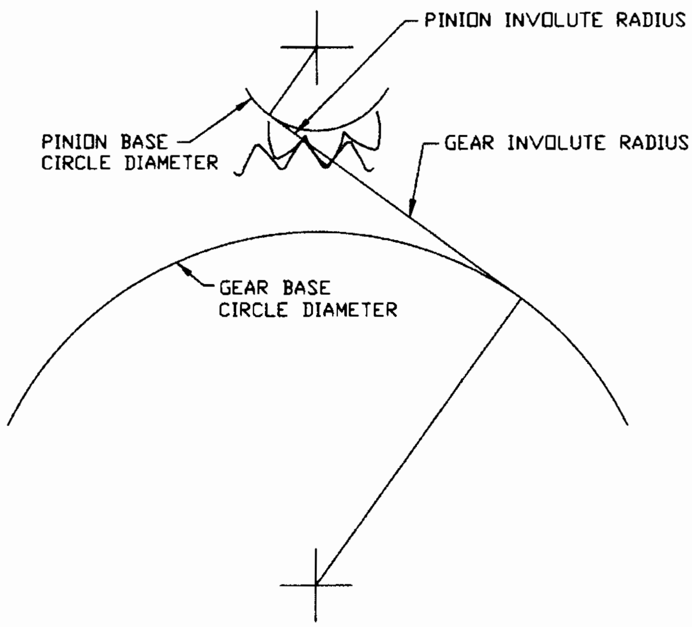


FIGURE 5 MEGAGEAR® 33.7° PRESSURE ANGLE GEARS

The AGMA rating of surface durability is based upon stress at the working pitch diameter. Table 1 shows significantly greater stress in the 25° pressure angle gears at the points of single tooth contact and start of active profile with corresponding great reductions in fatigue life at those points. The 33.7° pressure angle gears, however, have small variations in stress between the three points on the tooth profile and much greater fatigue lives.

Pictures Number 1 and 2 show a line of pits at the start of active profile. These two teeth are from a pinion that has a profile contact ratio of 1.3:1. There is no evidence of load sharing at the point of single tooth contact near the root diameter.

A very complex problem explained under the heading "Load Sharing Between Gear Teeth" is that an equal load division almost never occurs. The teeth shown in Picture Numbers 1 and 2 suggest that load division did not exist.

If we examine the surface compressive stress at the start of active profile of both pinions shown in Figures 1 and 2 and summarized in Table 1, we can see that because this point is closer to the base circle, the surface compressive stress is higher than that at the point of single tooth contact, or at the pitch diameter. This again reduces the surface fatigue life. Even though it is difficult to estimate the magnitude of load sharing between the teeth in order to obtain a better estimate of the surface fatigue, we can assume that the surface compressive stress is somewhere between the stress at point of single tooth contact and the stress at the start of active profile. The problem of calculating the surface compressive stress in the load sharing zone is further complicated by such enhancing feature as tooth crowning and tip relief.

Table 1 shows that the surface fatigue life is drastically reduced below the pitch diameter in standard gears. It is a simple concept. The calculations show that the further we examine the surface fatigue life towards the pinion root, the lower the life is. Figure 1 also shows that the closer a point on the involute curve is to the base circle, the smaller the involute radius is. Equation 1 in turn shows that the smaller the involute radius, the higher the surface compressive stress, and in turn the surface fatigue life is reduced.

This is irrefutable proof that the AGMA surface durability calculated based on the surface compressive stress and the pitch diameter can never be achieved. Only a fraction of that life will be realized.

Gear Design Practices

Until after World War II, most or all gears were manufactured with a 14.5° pressure angle. It is said that this angle was favored because it approximates the shape of a thumb. It is probably the shape that the rectangular teeth acquired through wear prior to the development of the involute profile.

After World War II, the design advanced to a 20° pressure angle. This is practiced almost universally around the world with some exceptions. In the United States, many companies have advanced the gear design to a 25° pressure angle. Very few if any gears made in other countries have a 25° pressure angle. The 25° pressure angle gear teeth are superior to the 20° pressure angle teeth because they offer a 7.5% increase in horsepower carrying capacity for the same surface compressive stress and the same surface durability. Equation 1 shows how to calculate the surface compressive stress for a set of gears in mesh. Equation 2 shows the torque relationship between 20° and 25° pressure angle gears. This is how the 7.5% increase in torque or horsepower capacity of a set of gears was derived.

Equation 2: HP Capacity Increase =

$$\left(\frac{COS\ 20}{COS\ 25}\right)^2 = 1.075$$

There are other advantages to the 25° pressure angle over the 20° pressure angle, such as reduced sliding, and an increase in oil film thickness between the teeth under load. Because the transverse cross section of the addendum of the 25° pressure angle is thinner than that of the 20° pressure angle, the bending stress in the addendum can be higher.

Because the dedendum of the 25° pressure angle tooth is increasingly thicker towards the root, the bending stress at the root is substantially lower than that of a 20° pressure angle tooth. For this reason the 25° pressure angle tooth has a higher bending fatigue life at the root. A good way to reduce the bending stress in the addendum is to reduce the profile contact ratio to nearly 1.0 by shortening the addendum.

This increase in surface compressive stress can be further avoided by making the gear tooth at a higher position on the involute profile. This necessitates an increase in the outside diameter of the gear with the smaller number of teeth that results in a corresponding increase in the root diameter. After such a modification, the bending stress in the addendum needs to be re-examined because the tooth becomes more nearly triangular in cross section..

Another way of reducing the bending stress in the addendum is to reduce the length of the addendum by reducing the gear outside diameter but still maintaining a profile contact ratio greater than one for spur gears. This method of reducing the length of the addendum has been practiced for many years and was called "Short Addendum" or the "Fellows Short Addendum". The "Short Addendum" automatically leads to a reduced outside diameter, which results in a reduced amount of material and reduced manufacturing time.

The practice of long addendum/short addendum, but still with a profile contact ratio greater than one, also resulted in an increased involute radius at the start of active profile, though the generating pitch diameter did not change. The surface compressive stress calculations, developed by the scientist Heinrich Rudolf Hertz (1857-1894), are based on the surface compressive stress between two cylinders, (See Figure 6 and Equation 1). The involute radius at any point of contact between the gear teeth can be calculated and consequently, the surface compressive stress and surface fatigue life can be determined. The six pairs of cylinders in Figure 6 make it evident that the larger the cylinders, the larger the area of contact under load and thus the surface compressive stress is reduced and surface fatigue life is increased.

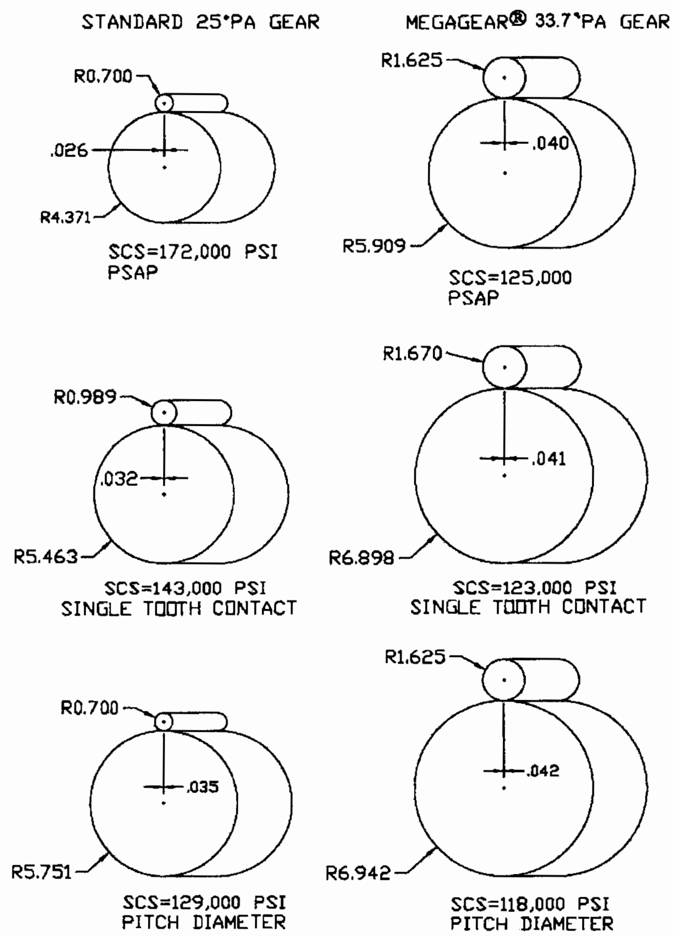


FIGURE 6 CONTACT WIDTHS AT DIFFERENT INVOLUTE RADII

For helical gears, the profile contact ratio may be less than one so long as the helical contact ratio compensates by being greater than one and together, the sum of the helical and profile contact ratios is greater than two.

The MEGAGEAR® and UNIMEGAGEAR® were slowly developed by PEM through intensive research and development and represents a very advanced gear technology. It was researched and evaluated by using them in thousands of heavy-duty gear boxes over many years. They provide a very high-power density, longer durability, and a higher efficiency.

A better way to eliminate undercut and to make a stronger tooth is to expand both the pinion and the gear diameters substantially past the point of undercut elimination. This will further reduce the bending stress as well as the surface compressive stress. However, care must be exercised to avoid making a tooth that is too pointed because the bending and shearing stress may become excessive at the tooth tip. Such an increase in the gear diameter will also reduce the surface compressive stress, resulting in an increased surface durability life because the tooth is positioned higher on the involute curve where the radius of curvature is larger. This simulates larger cylinders as shown on Figure 6.

If the surface compressive stress becomes so low that the gears are unnecessarily large, one or more teeth may be removed from the pinion and an equivalent number of teeth may be removed from the gear as dictated by the reduction ratio, to reduce the size of the gears until the surface compressive and bending stresses arrive at a desired magnitude for an adequate surface and bending durability. This is how MEGAGEARS® and UNIMEGAGEARS® are designed.

MEGAGEARS®

Since most gears are designed for a single purpose and single application, the power density and durability can be maximized by using the most suitable section of the involute profile for the pinion. Figure 3 shows that gear teeth can be constructed over a large section of the involute profile and not only at 20° or 25° where AGMA established standards. The long addendums are used in conjunction with pinions that have a small number of teeth in order to avoid tooth undercut that may lead to premature bending fatigue failure.

The MEGAGEAR® design process selects the best section of the involute for the pinion tooth simultaneously with the selection of the best involute section of the gear tooth because the pinion and the gear work together to transfer the power. It is the conjugate operation of the involute profile of both the pinion and the gear that determines the final operating durability characteristics.

It is desirable to move outward on the involute profile curve to use a section that is at the furthest distance from the base circle in order to achieve a tooth design that provides the best power density, durability, and efficiency. This is commonly referred to as the operating pressure angle of the gear tooth. The higher the operating pressure angle is, the further away the tooth is from the base circle. This property is not readily obvious, but the end result is that as the pressure angle increases at any point on the tooth, the distance from the base circle also increases.

UNIMEGAGEARS®

UNIMEGAGEARS® are a special case of MEGAGEARS® where the operating pressure angle is even higher on one flank when compared to the MEGAGEAR®. UNIMEGAGEARS® are used where the loads are predominantly in one direction. Loads can be applied in the opposite direction where the pressure angle is lower, but to achieve a balanced surface and bending durability, the loads on the lower pressure angle should be lower in magnitude or less frequent. Calculations can determine precisely the surface and bending fatigue lives for both flanks when the load magnitudes and their length of time are known for both directions of rotation.

The principle advantage is to increase the power density, thus reducing the weight of the gear box or transmission. This is achieved by having a longer flank on the higher pressure angle side of the gear tooth. This longer flank provides a larger area of load transfer between the teeth. Having a larger area of contact reduces the surface compressive stress, which can be increased to the original level simply by appropriately increasing the horsepower. This is how the UNIMEGAGEARS® enable an increased power density. See Figure 7.

UNIMEGAGEARS® have a higher power density than standard gears as well as MEGAGEARS®. They also have a thicker oil film under pressure between the gear teeth at the load transfer area, and are more efficient and less susceptible to scoring because of their inherent lower sliding.

When converting a set of MEGAGEARS® to UNIMEGAGEARS® a ten to twenty percent power density increase can be achieved while maintaining the same surface durability. When converting a standard set of 20° pressure angle gears to UNIMEGAGEARS®, an increased power density of 40 to 50 percent can be achieved. In most cases, bending fatigue life and efficiency are also improved. When the pinion surface compressive stress at the start of active profile is reduced to the lowest possible level, it will substantially increase the surface fatigue life of the pinion, and consequently of the entire gear box.

Figure 7 shows a UNIMEGAGEAR® tooth. Note that there are two base circle diameters for the two different pressure angle flanks. The higher pressure angle flank that is developed from the smaller base circle has a larger incline. This provides a larger area of load transfer between the teeth that enable transfer of a higher torque. The low pressure angle flank has a lower load carrying capacity. For this reason the UNIMEGAGEARS® should be used when the torque is predominantly in one direction.

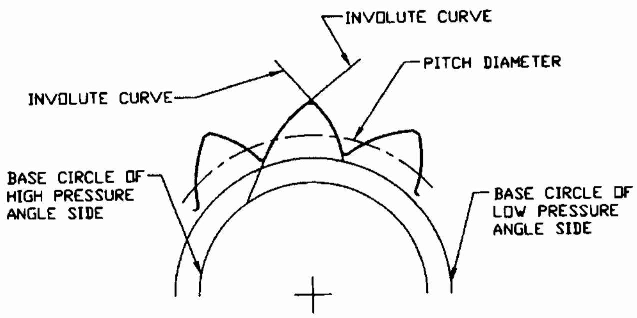


FIGURE 7 UNIMEGAGEAR®

Because the gear tooth involute radius of curvature is always larger than the pinion involute radius of curvature, the gear surface compressive stress at the start of active profile is always lower for gears with a higher ratio than 1:1.

Figure 8 shows the pinion and gear involute radius of curvature when the start of active profile contact is near the pinion root, as shown on A, and also when the gear tooth has contact at the start of active profile near the root, as in B. Note that in both cases, the length of the involute radius at the contact point of the gear is longer than that of the pinion.

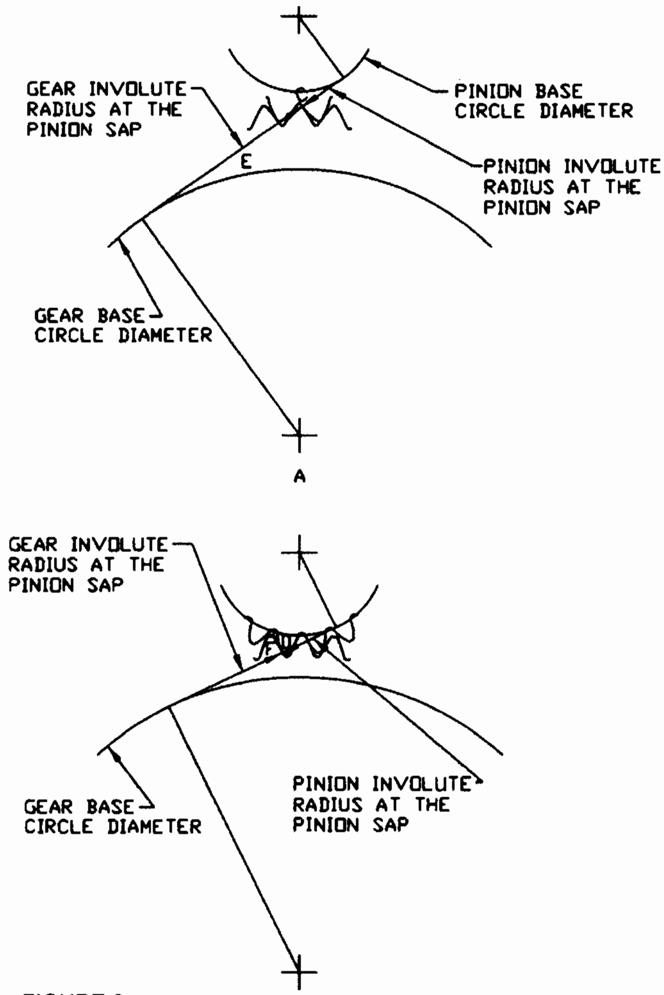


FIGURE 8
(A) Radii of Curvature When Contact is Near Root of Pinion Tooth
(B) Radii of Curvature When Contact is Near Root of Gear Tooth

If we assume that the gears in Figure 8, A and B are identical, by definition the pinion involute radius at the start of active profile, "C", is the shortest involute radius, therefore, the involute radius "C" determines the maximum surface compressive stress and consequently the surface fatigue life of the pinion. Note that Equation 1 shows how to calculate the surface compressive stress.

Superior Features of MEGAGEARS® And UNIMEGAGEARS®

Tables 2, 3 and 4 better illustrate the advantages of the MEGAGEARS® and the UNIMEGAGEARS®.

Table 2 Comparison of Surface Compressive Stresses and Life of 20° and 25° Pressure Angle Gears with MEGAGEARS® and UNIMEGAGEARS®

Run	1	2	3	4
Type of Gear	20°	25°	MEGAGEAR®	UNIMEGAGEAR®
SCS @ WPD (PSI)	199,000	183,000	170,000	165,000
Surface Fatigue Life [1]	1	1.75	2.86	3.49
Oil Film Thickness E-6	23-44	32-50	41-58	48-63
Pinion Max. Bending Stress	27,600	25,800	16,100	17,500
Gear Max. Bending Stress	40,000	34,200	26,500	30,700
Weight (lb)	222	222	222	222
Average Temp. Rise °F	48	35	29	24
Total Sliding ft/min	535	471	368	332
Horsepower	317	317	317	317
SCS@PCR 1 or SAP	193,000	182,000	177,000	165,000
Surface Fatigue Life	1.0	1.48	1.78	2.84

Table 3 Increase in Horsepower Using the Same Amount of Metal with Constant Surface Compressive Stress at the Working Pitch Diameter.

Run	1	5	6	7
Type of Gear	20°	25°	MEGAGEAR®	UNIMEGAGEAR®
Constant SCS@WPD	199,000	199,000	199,000	199,000
Weight (lb)	222	222	222	222
Horsepower	317	375	443	462
Increased Percent HP	0%	15%	28%	31%

Table 4 Comparison of Gear Weights of 20° and 25° Pressure Angle Gears with MEGAGEARS® And UNIMEGAGEARS®

Run	1	8	9	10
Type of Gear	20°	25°	MEGAGEAR®	UNIMEGAGEAR®
Constant SCS@WPD	199,000	199,000	199,000	199,000
Horsepower	317	317	317	317
Weight (lb)	222	188	159	153
Weight Reduction %	0%	15%	28%	31%

The calculations In Table 2 were made with all gear sets having the same reduction ratio, same face width, same center distance, and all of them having the same material weight. Note that the surface compressive stress at the working pitch diameter decreases as we read the table from left to right. The lower the surface compressive stress is, the higher the surface fatigue life becomes. The surface fatigue life based on surface compressive stress is calculated by using Equation 3;

Equation 3: Surface Fatigue Life = $\left(\frac{SCS_1}{SCS_2}\right)^{6.666}$

It is important to note in Table 2 that the surface fatigue life of MEGAGEARS® is nearly three times greater than that of the 20° pressure angle gears. The oil film between the gear teeth of the MEGAGEARS® and UNIMEGAGEARS® is also substantially thicker than the standard 20° and 25° gears. A longer surface fatigue life is expected when the separation between the gear teeth is so great that the opportunity for the hydrostatic load transfer is achieved.

The Timken Company technical papers entitled:

“Modifying the Lambda Ratio to Functional Line Contacts”,

“The Use of Elastohydrodynamic Lubrication in Understanding Bearing Performance”, and

“Using the Modified Lambda Ratio to Advance Bearing and Gear Performance”,

show that it is desirable to have an oil film thickness that is 2.4 times or even greater than the height of the surface asperities.

The average oil temperature rise decreases as we move from Column 1 to 4 for the simple reason that the amount of sliding decreases, as shown in the next row. In general, the surface compressive stress is calculated at the pitch diameter only, but it is always higher at the start of active profile on the gear with the smaller number of teeth, because the involute radius of curvature at this point is the smaller. The calculation in Table 2 shows that MEGAGEARS® have a surface fatigue life that is nearly double the 20° pressure angle gears. Likewise, the UNIMEGAGEARS® have a surface durability at the start of active profile nearly triple that of the 20° pressure angle gears.

Standard gear 23 teeth, 4.233 NDP, 20° PA, 30° HA
1264 HP at 3000 RPM, 50 million load cycles
SCS@PCR=231,000 PSI, 6.753 O.D.

Picture 3

Megagear® 19 teeth, 4 NDP, 35° PA, 30° HA
1264 HP at 3000 RPM, 50 million load cycles
SCS@SAP=186,000 PSI, 6.813

Picture 4

Megagear® 19 teeth, 4 NDP, 35° PA, 30° HA
1686 HP at 3000 RPM, 50 million load cycles
SCS@SAP=215,000 PSI, 6.813 O.D.

Picture 5

Picture Numbers 3, through 5, are of gears tested at the Design Unit Gear Testing Laboratory at the University of Newcastle upon Tyne in England. The three different tests were performed identically as much as technically possible. The steel used was from the same pour, courtesy of The Timken Company. All tests were stopped at 50 million load cycles. The tests were conducted at constant 3,000 rpm, constant oil temperature, and constant torque. The objective was to initiate scoring as shown on Picture 3, which is a 20° pressure angle helical gear. When a set of MEGAGEARS® were tested in an identical manner as shown on Picture 4, the gear did not develop any score marks. The darker line near the root diameter is polish created by the sharp intersection corner between the top land and the flank. A larger tip relief can reduce this polish.

In order to determine the additional power capacity of the MEGAGEARS®, another test was done with an additional 33 percent horsepower. At this power level scoring began to appear as shown on Picture 5.

Because these test are highly accurate and reliable it is proven that the MEGAGEARS® have a power density that is up to 33 percent higher than that of the 20° pressure angle gears.

Table 2 shows that at a constant horsepower, the surface fatigue life can be increased drastically when comparing the MEGAGEARS® and UNIMEGAGEARS® with 20° or 25° pressure angle gears. The table lists other important benefits of MEGAGEARS® and UNIMEGAGEARS® compared to 20° or 25° pressure angle gears such as: reduced bending stress that increases the bending fatigue life, increased oil film thickness between the teeth at load transfer area, and reduced oil film temperature rise due to reduced sliding.

The calculations in Table 3 are based on holding the center distance, reduction ratio, and face width fixed to maintain a constant surface compressive stress at the working pitch diameter. This shows that the MEGAGEARS® have 28 percent higher power density and the UNIMEGAGEARS® have 31 percent higher power density.

Table 4 is similar to Tables 2 and 3 where the center distance and reduction ratio remain identical to all other calculations, except that in this table the face width is varied in order to maintain a constant surface compressive stress at a constant horsepower at the working pitch diameter. The objective here is to show the weight savings possible when shifting the design from 20° to 25° to MEGAGEARS® or to UNIMEGAGEARS®.

An exact universal ratio of power density or surface fatigue life increase for MEGAGEARS® or UNIMEGAGEARS® over standard gears can not be established because each gear set can have some geometry variation. For example, if we reduce the number of teeth by 50% as well as reduce the diametral pitch by 50% to maintain the same pitch diameter, the tooth becomes thicker and the whole depth larger. The effect of the increased tooth thickness is to reduce the bending stress, which results in increased bending fatigue life. The increased whole depth, without any addendum modification will result in an increased surface compressive stress because the start of active profile is closer to the base circle diameter. This is not desirable because the surface fatigue life is reduced. In this case, the surface compressive stress at the start of active profile can be reduced by moving the pinion tooth radially on the involute curve as far away as possible, as is the practice with MEGAGEARS® and UNIMEGAGEARS®.

Design variables such as diametral pitch, pressure angle, addendum modification, and number of teeth determine the position of the pinion start of active profile, or point of single tooth contact, and it will vary from one design to another.

This is the point on the tooth that determines the maximum surface fatigue stress and resulting minimum surface fatigue life. For this reason it is necessary to calculate the pinion surface fatigue life for every set of gears at the start of active profile or at the point of single tooth contact.

Load Sharing Between Gear Teeth

When spur gears rotate, the load is transferred instantaneously from one tooth to another along the full face width of the tooth on a line that is parallel with the gear centerline. The instant that the forces change from load sharing to no load sharing, the force magnitude increases again along the full length of the tooth. The tooth deflection has been rapidly relieved on the unloaded tooth and that created a slight impact on the tooth that now carries the full load. On accurate ground gears that operate at fairly low speeds, the position of transition from load sharing to no load sharing can be identified as a line on the gear teeth after some time of usage. Spur gears that are heavily loaded make a high frequency whining noise that is similar to the sound of a tuning fork when the pinched fork is suddenly released. Because of the sudden load release of one tooth and sudden increase in force on the tooth that now carries the load at the transition point, it may be stated that a mild shock load occurs. In most cases, because of the oil film thickness, no damage occurs. However, inevitably the force is somewhat higher than the force that corresponds to the full load.

Helical gears that have a combined profile and helical contact ratio sum that is greater than two are superior to spur gears because the load is transferred gently from one tooth to another due to the fact that two teeth will always share the load. An incoming tooth begins to carry the load before the outgoing tooth is relieved of load. The tooth deflection of the helical gears is more complex because the line of load is on a diagonal; therefore, the tooth deflection occurs instantaneously along the entire radial length of the tooth. The line of load moves axially as the gears rotate. The deflection of a helical gear tooth is less than that of an equivalent spur gear tooth because the line of load, being in the diagonal direction, allows the metal adjacent to the line of load, in the axial direction, to resist deflection shown on Figure 9.

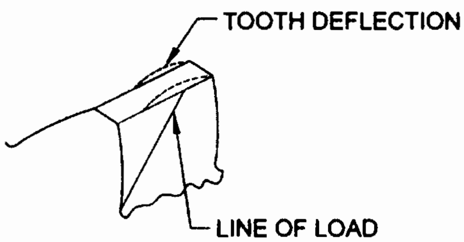


FIGURE 9 Deflection of Helical Gear Tooth

If the deflection is lower, the bending stress is also lower. Because spur gears have an axial line of load along the tooth, they operate with tooth deflection. This spur gear deflection generates noise but it can also initiate vibrations, particularly in a mechanism where a natural frequency of vibration is near the gear teeth meshing frequency.

Helical gears with a helical contact ratio that is greater than 1.0 transfer the load to the next tooth before the load is removed from the adjacent tooth. This feature makes the helical gear quieter and less likely to induce vibrations. For these reasons, helical gears are superior to spur gears.

It is often assumed that equal load sharing between gear teeth can take place during operation for gears that have a profile contact ratio greater than one. Helical gear teeth similarly have additional load sharing over spur gears simply because the helix angle offers a helical contact ratio. This takes place regardless of the profile contact ratio. Upon close examination it will be shown that equal load sharing between gear teeth at the portion of the teeth that operate with a greater contact ratio than one can almost never take place. Some load sharing may occur, however, very seldom if ever will this load sharing be nearly equal between the pinion teeth and the gear teeth for the following reasons:

1. Manufacturing accuracies are such that the imperfections of concentricity, tooth spacing, profile error, helix error, thermal expansion during operation, shaft deflection, housing deflection, bearing bore position, bearing clearances, and wear with time are such that cumulatively very seldom, if ever, would allow equal load sharing between the pinion and the gear teeth as they rotate to transmit power.

2. The teeth on gears of unequal number have different bending spring rate characteristics. If we assume that the gear teeth form short cantilever beams, then when the number of teeth vary between two gears in mesh, particularly at higher reduction such as in a 4:1 or 10:1 reduction ratio, the cross section of the teeth is substantially different. Therefore, the forces required to deflect the teeth in an equal amount would be different in magnitude between the pinion teeth and the gear teeth. The spring rate of the pinion tooth is different from the spring rate of the gear tooth. For the pinion, the deflection at the tip of the tooth would be different than the gear tooth deflection near the tip of the tooth at the instant of time when load sharing occurs often. The deflection of the gear tooth is higher than the deflection of the pinion tooth because the pinion tooth is constructed on a smaller base circle diameter than the gear tooth. This gives the pinion a thicker cross section. See Figure 10. Because the spring rate of the pinion and gear teeth are unequal, the deflection at the tip of the pinion tooth is not equal to the deflection at the tip of the gear tooth. Consequently, the bending fatigue, operating characteristics of the pinion teeth and gear teeth would be different.

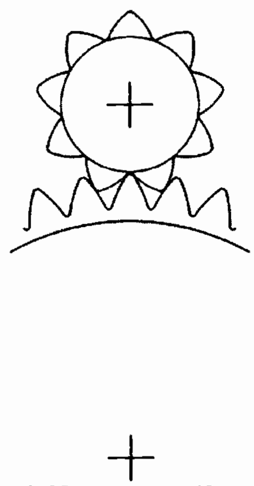


FIGURE 10 Comparison of Pinion and Gear Tooth Thickness

3. The amount of tooth surface compressibility also has an effect on the tooth deflection. Because the pinion tooth radius of curvature is different than the gear tooth radius of curvature, it results in different deflections in the elastic zone.

4. Even the oil film thickness has an influence on the load distribution between teeth above one profile contact ratio. The thicker the oil film is, the wider the area of load transfer is. A hydrostatic load transfer is ideal because it eliminates mechanical contact that is associated with higher localized forces, higher stresses, and lower fatigue life.

5. The load sharing is also affected by the modern gear tooth grinding machines that are able to provide tip relief, which is to grind an additional several tenths of one thousandths of an inch near the tooth tip and fading to zero towards the pitch diameter.

6. Gear teeth have pure rolling at the pitch diameter only and sliding increases away from the pitch diameter in either direction. Surface compressive stress also varies along the tooth. Deflection that allows load sharing between the teeth also varies from the tip of the tooth towards the root diameter, decreasing in magnitude from the tip until there is hardly any deflection at the start of active profile near the root diameter. Sliding in combination with high forces can create wear. Because the sliding velocities are different in the radial direction at any point on the tooth and because the surface compressive stress also varies, the amount of wear on the tooth will have an effect on the tooth deflecting characteristics of the pinion and gear teeth. However, this is much more complex to calculate. To further complicate the estimation of wear, helical gears have a different tooth deflection at any given instant along the tooth because the line of load transfer is on a diagonal line along the tooth. On spur gears, the line of load is in a straight line parallel to the axis of rotation. The oil film thickness and the extreme pressure (EP) additives also have a major influence on the rate of wear. As wear progresses, the load sharing characteristics of the gear teeth are changing. It stands to reason that on gears with a hunting tooth, where each tooth in each gear meshes with each tooth on the mating gear, the wear is more uniform and may lead to more uniform load transfer characteristics. Therefore, the load sharing, even though it will not necessarily be equal in magnitude, should be expected to improve with wear. In such gears, with a more uniform wear characteristic, the magnitude of forces between teeth during load sharing will become more uniform. This results in elimination of the high localized forces that accelerate fatigue and wear. The highest amount of wear that results in metal removal should be away from the pitch diameter and increasing in magnitude, reaching a maximum at the start of active profile on the pinion.

FATIGUE LIFE: Because a nearly equal force magnitude division cannot occur a majority of the time between the pinion teeth and the gear teeth, the fatigue life is highly affected for the simple reason that when the force magnitude changes, it changes linearly by the first power. The surface durability fatigue is to the power of 3.33 and the bending durability fatigue is to the power of 6.

Note: Table 5 shows a decrease in fatigue life magnitude based on various levels of force differences. However small the differences may be, even a 10% difference still creates a very high reduction in the fatigue life, in bending or in surface spalling.

(Refer to Table 5) For example, it may happen that the tooth deflection is twice as big on the gear tooth as on the pinion tooth because the pinion tooth is stiffer and has a higher spring rate in bending due to the fact that it is constructed on a smaller base circle. If we use the steel fatigue property of surface durability as used in bearings, and we assume that the resulting force is twice as big, then $2^{3.333}$ equals 10. Therefore, when load sharing exists, the one member that has twice as high a force as the other member, the surface fatigue of the member with the higher force is reduced to ten percent life. The same thing happens with the bending stress. If the member that has twice as high a force happens to have a bending stress that is also twice as high, the reduction in bending fatigue life is to 1/64 or 1.5 percent. This reduction in life is obtained by raising $2^6 = 64$.

TABLE 5 Variation in Fatigue Lives With Change In Load

Load Sharing Per Tooth	Load	Durability		Load	Durability	
Percent	Percent Load	Surface Compression	Bending	Percent Load	Surface Compression	Bending
50/50	50	1.00	1.00	50	1.00	1.00
60/40	40	1.84	2.99	60	0.54	0.33
70/30	30	3.07	7.53	70	0.33	0.13
80/20	20	4.79	16.77	80	0.21	0.06
90/10	10	7.09	34.01	90	0.14	0.03
100/00	0	10.07	64	100	0.10	0.02

Gear Box Efficiency

There are four sources of heat generation in a gear box:

1. The sliding friction at the gear teeth.
2. The lubricant movement forced by the gears and bearings.
3. Friction at the bearings.
4. Sliding friction at the oil seals.

The friction at the bearings and oil seals is very small as compared to the friction at the gear teeth and at the oil movement. For this reason, it can be discounted as a substantial source of heat generation, as it is only a marginal heat source.

The heat generated by the oil can come from several sources. One is the friction of the oil molecules as they slide past each other. The second is the impact of high velocity oil streams as they contact case walls or other surfaces. At instances where the gear mesh is lubricated with pressure oil at the ingoing mesh, most of the oil will be forced sideways by the gear teeth. The speed with which the oil moves depends on the pitch line velocity of the gears and the helix angle. Under similar operating conditions, spur gears force the oil in the space between the teeth faster than helical gears because the helical gear teeth have a larger angle of rotation for the space between the teeth than the spur gears. For this reason, the oil velocity at case wall impact of spur gears is higher than that of helical gears; therefore, spur gears generate more heat and are less efficient.

Heat is generated at the gear teeth due to sliding friction. The quantity of friction heat is proportional to the force at the teeth, therefore it is proportional to the horsepower. The horsepower loss at the gear teeth is about one percent per mesh. The friction loss at the bearings, oil seals, and oil churning can add up to another ¼% loss per gear mesh.

The further away the point of contact is from the pitch diameter, the larger the sliding velocity is. The amount of heat generated due to sliding is determined by the law of physics, which is ***work equals force times distance***.

Our experience is that spur or helical gears that have a pitch line velocity below 1,500 ft/minute are not generating any substantial amount of heat due to lubricant movement or internal oil friction. Some variations will exist depending on the external surface of the case and its heat exchange properties.

The efficiency of the MEGAGEARS® and the UNIMEGAGEARS® is higher due to two factors. Number one is that due to their higher power density, the pitch diameters are smaller, resulting in lower pitch line velocities. The lower the pitch line velocity, the less the heat generated. Number two is the reduction in heat generation is due to the inherent property of MEGAGEARS® and UNIMEGAGEARS® having a lower total sliding than standard gears.

CONCLUSION

MEGAGEAR® technology offers the advantage of increasing the power density of gears. UNIMEGAGEAR® technology offers the possibility of further increasing the power density over the MEGAGEAR®, but only for applications where the load is substantially greater or the same load but for a shorter amount of time in one direction than the other direction.

Other improved features of both MEGAGEARS® and UNIMEGAGEARS® are also present, such as reduced sliding which results proportionally in reduced lube oil temperature rise, reduced energy loss, increased efficiency, increased oil film thickness between the gear teeth, and a wider area of contact between the teeth.

Mechanical load transfer at the high points of the asperities will have higher stress concentrations that results in premature surface fatigue. The increase in the oil film thickness is of great benefit because Timken's technical papers:

"Modifying the Lambda Ratio to Functional Line Contacts";

"The Use of Elastohydrodynamic Lubrication in Understanding Bearing Performance"; and

"Using the Modified Lambda Ratio to Advance Bearing and Gear Performance",

showed that when the oil film thickness exceeds the height of asperities by more than 2.4 times[1], hydrostatic load transfer begins to take place and mechanical load transfer diminishes or completely disappears.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

Equation 1: PRD = PPD – 2HAD + ΔAD

Equation 2: GRD = GPD – 2HAD - 2ΔAD

Equation 3: BCD = GPD x COS(GPA)

Equation 4: GPD = T/GDP x COS(HA)

DEFINITIONS:

AGMA: American Gear Manufactures Association

AD: Change in gear addendum length

BCD: Base Circle Diameter

COS: Cosine

EP: Extreme Pressure

GDP: Normal Generating Diametral Pitch

GGPD: Gear Generating Pitch Diameter

GIR: Gear Involute Radius

GPA:	Normal Generating Pressure Angle
GRD:	Gear Root Diameter
HA:	Helical Angle at the Generative Pitch Diameter
HAD:	Cutter Addendum
HP:	Horsepower
OD:	Outside Diameter
PA:	Pressure Angle
PEM:	Power Engineering and Manufacturing, Ltd.
PIR:	Pinion Involute Radius
PGPD:	Pinion Generating Pitch Diameter
PRD:	Pinion Root Diameter
SAP:	Start of Active Profile
T:	Number of teeth
WPD:	Working Pitch Diameter

SYMBOL	DESCRIPTION	UNITS
π	Pi = 3.14159	
ν	Poisson's Ratio = 0.30	
E	Modulus of elasticity = 30*10^6	psi
BR	Base Radius	in
FW	Face Width	in
LLOA	Length of the line of action	in
PCR	Profile contact ratio	.
r1	Radius of curvature	in
r2	Radius of curvature	in
SCS	Surface compressive stress	psi
T	Torque	in-lb