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INTRODUCTION TO GEAR DESIGN

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ABSTRACT

The purpose of this article is to provide the design formulas and acceptable stress levels so that calculations of the gear surface compressive and bending stresses at which the gears will operate in a known application can be made. It will be necessary to know the complete gear information for all gears, such as number of teeth, diametral pitch, pressure angle, gear width, type of material, and type of heat treatment, if any.

The surface compressive and bending stresses are based on American Gear Manufacturers Association (AGMA) formulas and tables, and the applications stress limits are based on Machine Design recommended safe stress levels and authors past experience. By combining the proper number of teeth with the proper size tooth, the proper pressure angle, the proper material and heat treatment, a durable gear box can be designed to offer a long life under any operating conditions.

IT IS THOUGHT that scientist Euler was the first one to recognize that the involute profile form of the gear tooth provides constant angular velocity. It was not until the beginning of this century that this tooth form became widely used and now it is almost exclusively used for transfer of heavy or light loads at low or high angular velocity.

A simple development of the involute profile is obtained by unwinding a wire off a cylinder and having a pencil tied to the end of the wire and drawing a line on a sheet of paper while unwinding the wire as illustrated on Figure 1. In this case the cylinder and the paper are stationary, and the pencil moves. Another way of describing an involute profile is by wrapping a belt in a figure "8" on two parallel cylinders, attaching a pencil on the belt and rotating the cylinder on a sheet of paper that is attached to one cylinder as shown on Figure 2. If two pencils are attached on the points A and B on the belt and cylinder L has a sheet of paper with diameter C attached to itself and is being rotated in both directions, the pencils will draw two curves from the paper outside diameter to the cylinder diameter D. These curves have an involute profile and can be the profile of a gear tooth. If the belt had zero thickness so that the drawing pencil can travel on diameter D or E, then the D and E diameters become the gear base circle diameters of the respective cylinders L and R.

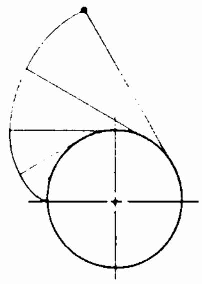

FIGURE 1

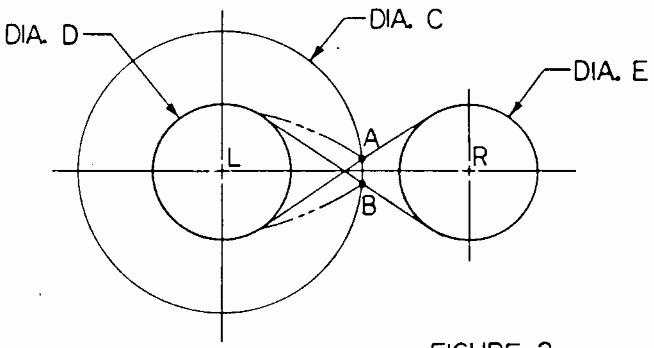

FIGURE 2

DESIGN PROCEDURE:

To better illustrate how to design a set of gears an example of two helical gears will be calculated step by step with the writing of the basic formulas. The pinion will have 17 teeth and the gear will have 52 teeth. This pair of gears provides a reduction ratio of 52 divided by 17 = 3.059. The equations can be used interchangeably between spur and helical gears since the helical angle of a spur gear is zero and the cosine of zero degrees is one. Even though the calculating of one gear could illustrate the utilization of the gear design equations, two gears were selected for calculations to show how to modify the addendums to eliminate undercut on the pinion and because contact ratio and operating stresses require the use of two gears to achieve power transfer.

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Once the gear reduction ratio was made, one can proceed to select the diametral pitch, normal pressure angle, and the helical angle. This is a trial and error selection based on availability of parts or tooling and the hope that this first selection is suitable for the required performance. If the selection is not suitable, the calculated results may be compared with the allowable stresses shown on Table 1 for the respective materials and a second set of calculations may be initiated making the gears larger or smaller as desired to obtain the recommended operating stress levels.

Table 1

Material	Type of Heat Treatment	Hardness	Tensil Strength (PSI)	Allowable Surface Compressive Stress	Allowable Bending Stress
Low Carbon Steel	Carborize & Case Harden	Rc 60	325,000	200,000	50,000
Low Carbon Steel	Carborize & Case Harden	Rc 55	285,000	180,000	45,000
Low Carbon Steel	Carborize & Case Harden	Rc 50	245,000	155,000	40,000
Low Carbon Steel	Carborize & Case Harden	Rc 45	212,000	132,000	33,000
Low Carbon Steel	Carborize & Case Harden	Rc 40	186,000	115,000	28,000
Med. Carb. Steel	Full Hardened	BHN 440	223,000	140,000	30,000
Med. Carb. Steel	Full Hardened	BHN 360	180,000	110,000	26,000
Med. Carb. Steel	Full Hardened	BHN 300	150,000	90,000	22,000
Med. Carb. Steel	Full Hardened	BHN 240	116,000	70,000	17,000
Med. Carb. Steel	Full Hardened	BHN 180	80,000	50,000	12,000

Before the equations are written it is necessary to establish the symbols of the variables to be used. The following is a list of definitions of the symbols. Some of these definitions are illustrated in Figure 3.

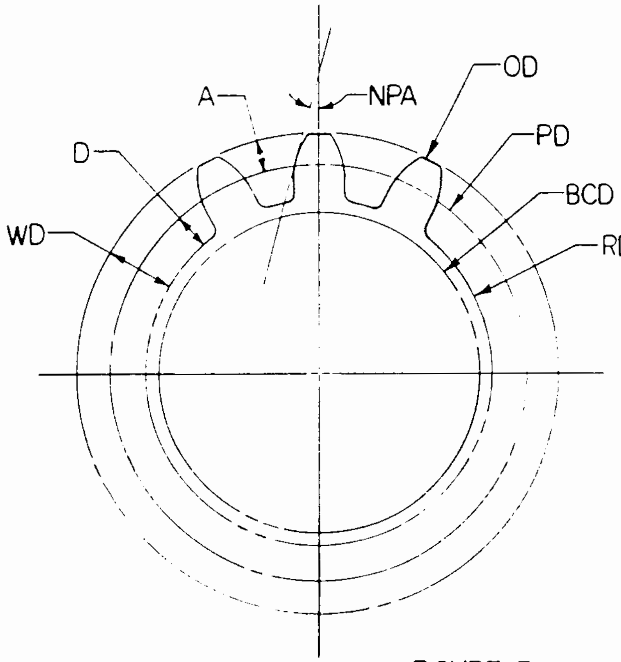

FIGURE 3

A .... Addendum. The length of tooth from the nominal pitch diameter to the outside diameter.
BCD .... Base circle diameter.
BL ...... Backlash is the operating clearance between the teeth.
BP ...... Base pitch.
CL ...... Clearance between the tooth tip and root diameter of the mating gear.
CP ...... Circular pitch.
D .... Dedendum. The length of tooth from the nominal pitch diameter to the root diameter.
DR ...... Change in pinion radius to eliminate undercut.
DTTT .... Change in transverse tooth thickness resulting from change in pinion addendum to eliminate undercut.
DU ..... Diameter where undercut begins.
E .... Modulus of elasticity (30×10^6 PSI for steel)
FW ..... Gear face width.
G .... Gear. The gear is the driven gear.
GBR .... Gear base radius.
GOR .... Gear outside radius.
GNPD ... Gear nominal pitch diameter.
GTTT .... Gear transverse tooth thickness.
HA ...... Helix angle. For spur gears use a helical angle of zero degrees.
HCR .... Helical contact ratio.
HP ...... Horsepower.
L .... Lead. A point anywhere on the tooth that is found again when the tooth makes a 360 degree spiral on the gear.
NDP .... Normal diametral pitch. This indicates how many gear teeth are on a spur gear that has a one inch pitch diameter.
NPA ..... Normal pressure angle. This is the angle that a tangent line on the tooth at the pitch diameter forms with a line that joins the center tooth thickness with the gear center.
NPD .... Nominal pitch diameter.
OD .... Outside diameter.
P ...... Pinion. The pinion is the driving gear.
PBR .... Pinion base radius.
PCR .... Profile contact ratio.
PNPD .... Pinion nominal pitch diameter.
POR .... Pinion outside radius.
R ...... Corner radius between hob outside diameter and the pressure angle face. See Figure 4.
RD ..... Root diameter.
RPA ..... Pressure angle in the plane of rotation at the nominal pitch diameter.

RPM .... Gear speed expressed in revolutions per minutes.
RU ...... Length of radial undercut.
SB ...... Bending stress.
SC ...... Surface compressive stress.
T .... Torque.
TE ...... Number of teeth.
TL ...... Tangential load.
VCF ..... Velocity correction factor.
WD ..... Whole depth. The radial length of the tooth from the root diameter to the outside diameter.

It is most desirable to first establish the torque and reduction ratio at which the gears have to operate so that when calculating a matched set of gears the stress levels are established.

To determine if this set of gears is adequate, the calculated stress level has to be compared with the permissible stress levels shown on Table 1. If the gears have stress levels that are too high, larger gears must be selected so that adequate life is attained. The first approach of selecting gears with increased load carrying capacity should be made by increasing the gear width because increase in the cost of material is proportional to the width. When it is necessary to increase the gear load carrying capacity by increasing the diameters it should be taken into account that the pitch line velocity increases and the cost of material increases with the square of the diameter increase.

The **Nominal Pitch Diameter** is an invisible circular line slightly above the mid-height of the tooth. At this point pure rolling exists when the gears rotate. Surface sliding is generated and increases as the load lines are farther and farther away from the nominal pitch diameter either towards the outside diameter or the root diameter.

$$NPD = \frac{TE}{NDP \times COS(HA)} \qquad (1)$$

$$\text{Pinion } NPD = \frac{17}{8 \times COS(15)} = 2.200 \text{ in.}$$

$$\text{Gear } NPD = \frac{52}{8 \times COS(15)} = 6.729$$

The **Center Distance** is determined by the gear diameters and it is calculated as follows:

$$CD = \frac{PNDP + GNPD}{2} \qquad (2)$$

$$CD = \frac{2.200 + 6.729}{2} = 4.464 \text{ in.}$$

The **Addendum** is the distance from the nominal pitch diameter to the outside diameter. Its length is customarily the reciprocal of the normal diametral pitch. It can be any length so long as it yields a contact ratio greater than one under the worst tolerance accumulation conditions.

$$\text{Addendum: } A = \frac{1}{NDP} \qquad (3)$$

$$\text{Pinion } A = \text{Gear } A = \frac{1}{8} = .125 \text{ in.}$$

The **Dedendum** is the distance from the nominal pitch diameter to the root diameter. It must be greater than the addendum to provide clearance between the root diameter and the outside diameter of the mating gear. It is customarily 1.157 times the length of the addendum.

$$\text{Dedendum: } D = \frac{1.157}{NDP} \qquad (4)$$

$$\text{Pinion } D = \text{Gear } D = \frac{1.157}{8} = .145 \text{ in.}$$

Clearance is the space between the tip of the tooth and the root diameter of the mating gear.

$$CL = \frac{.157}{NDP} \qquad (5)$$

$$\text{Pinion } CL = \text{Gear } CL = \frac{.157}{8} = .020 \text{ in.}$$

It should be noted that this clearance is for gears without backlash. Since all gears must have backlash, it follows that the above clearance becomes larger by an amount that is equal to the cutter radial infeed that generates the backlash.

The **Root Diameter** is the sum of the nominal pitch diameter minus two dedendums. In reality, the root diameter is slightly smaller because the gear teeth are slightly thinned to provide operating clearance. The reduction in root diameter corresponds to the amount of backlash provided by each gear. The dimensional reduction in the root diameter can be calculated but it has no significant effect on the gear bending strength and no effect at all on the surface compressive stress.

$$\text{Root Diameter} = NPD - 2D \qquad (6)$$

$$\text{Pinion } RD = 2.200 - 2 \times .145 = 1.910 \text{ in.}$$

$$\text{Gear } RD = 6.729 - 2 \times .145 = 6.439 \text{ in.}$$

The **Lead** is a point anywhere on the tooth that is found again in the axial plane when that tooth makes a 360 degree spiral on the gear. A change in the manufactured lead from the theoretical lead indicates a change of the helix angle that may result in uneven load distribution across the tooth face. When the lead error per tooth is equal and in the same direction for both the pinion and the gear, uniform load distribution is achieved even though a lead error exists.

Because spur gears have no helical angle the lead is infinite. A lead inspection of a spur gear only shows how well the tooth was manufactured in the axial plane and indicates how good a load distribution will be achieved.

$$\text{Lead: } L = \frac{NPD \times 3.14}{TAN(HA)} \qquad (7)$$

$$\text{Pinion Lead} = \frac{2.200 \text{ X } 3.14}{\text{TAN}(15)} = 25.794 \text{ in.}$$

$$\text{Gear Lead} = \frac{6.729 \text{ X } 3.14}{\text{TAN}(15)} = 78.895 \text{ in.}$$

The **Normal Tooth Thickness** at the nominal pitch diameter for an unmodified gear is:

$$\text{NTT} = \frac{3.14}{2 \text{ X NDP}} \qquad (8)$$

For the gears calculated in the above example, the normal tooth thickness is:

$$\text{Pinion NTT} = \text{Gear NTT} = \frac{3.14}{2 \text{ X } 8} = .196$$

The tooth should be thinned by several thousandths of one inch to provide operating backlash. Note that for a spur gear the normal tooth thickness at the nominal pitch diameter is equal to the transverse tooth thickness at the nominal pitch diameter.

The **Transverse Tooth Thickness** is measured at the nominal pitch diameter and it is in the plane of rotation. It should be noted that even though the normal tooth thickness remains the same for any helical angle, the transverse tooth thickness increases as the helical angle increases. This dimension is used in calculating the dimension over rolls for manufacturing. This tooth thickness should be slightly reduced so that operating clearance on the non-loaded face of the tooth is provided. If operating clearance does not exist, radial loads can be developed that are many times higher than the designed loads and premature failures will result.

The transverse tooth thickness for a helical gear set where both gears have equal tooth thickness at the nominal pitch diameter can be calculated as follows:

$$\text{TTT} = \frac{\text{NPD X } 3.14}{2 \text{ X TE}} \qquad (9)$$

$$\text{PTTT} = \text{Gear TTT} = \frac{2.200 \text{ X } 3.14}{17 \text{ X } 2} = .203 \text{ in.}$$

The tooth thickness should be reduced by several thousandths to provide operating backlash.

The tooth thickness should be made nearly equal for both gears so that the tip thickness does not become too small. Gear sets of unequal tooth thickness can be designed but this procedure is beyond the scope of this paper.

The **Pressure Angle in the plane of rotation at the nominal pitch diameter** is the same as the normal pressure angle for spur gears, but it is larger than the normal pressure angle for helical gears. It is calculated as follows:

$$\text{RPA} = \text{ARCTAN (TAN(N1PA)/COS(HA))} \qquad (10)$$

$$\begin{aligned}\text{Pinion RPA} &= \text{Gear RPA} \\ &= \text{ARCTAN (TAN(20)/COS(15))} \\ &= 20.647 \text{ deg}\end{aligned}$$

The **Diameter Where Undercut Begins** is calculated below. It is desirable to have this diameter smaller than the root diameter so that the gear teeth will be manufactured without undercut. Undercut reduces the bending strength of the tooth leading to premature fatigue failure, and it also may reduce the profile contact ratio if the undercut extends into the active profile.

$$\text{DU} = \text{NPD X COS}^2\text{(RPA)} - 2 \text{ X} \left(\frac{.160}{\text{NDP}}\right) \text{X (1 - SIN(RPA))} \qquad (11)$$

$$\text{POU} = 2.200 \text{ X COS}^2 \text{ (20.647) -2 (.160/8) X (1-SIN(20.347))} = 1.900$$

The amount of **Radial Undercut** is:

$$\text{RU} = \frac{\text{RD - DU}}{2} \qquad (12)$$

$$\text{Pinion } \triangle \text{ RU} = \frac{1.910 - 1.900}{2} = .005$$

The pinion will have a small amount of undercut. This small amount of .005 in. on an 8 normal diametral pitch gear will not have a measurable effect on the tooth strength.

Whenever it is desired to eliminate the undercut it can be done by increasing the pinion addendum by the amount of radial undercut and reducing the gear addendum by an equal amount. It is also necessary to modify the tooth thickness so that the pinion does not become too weak and to avoid the possibility that the tip may become pointed. The pinion transverse tooth thickness may be increased by the following formula (also see Figure 4):

$$\text{DTTT} = \frac{\text{DR X TAN(NPA) X } 2}{\text{COS(HA)}} \qquad (13)$$

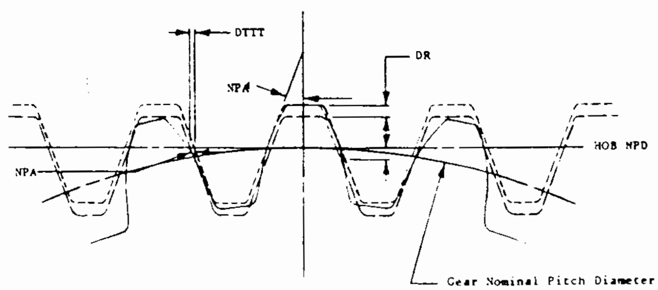

FIGURE 4

The **Gear Transverse Tooth Thickness** may be determined by subtracting the pinion tooth thickness from the circular pitch.

$$\text{CP} = \frac{\text{NOP X } 3.14}{\text{TE}} \qquad (14)$$

$$\text{CP} = \frac{2.200 \text{ X } 3.14}{17} = .406 \text{ in.}$$

$$GTTT = CP - PTTT \quad (15)$$

$$GTTT = .406 - .203 = .203 \text{ in.}$$

In this example the pinion tooth thickness equals the gear tooth thickness because they have not been modified.

The **Pitch Line Velocity** is a function of pitch diameter and speed. It is calculated in feet per minute as follows:

$$PLV = NPD \times 3.14 \times RPM/12 \quad (16)$$

$$PLV = 2.200 \times 3.14 \times 2000/12 = 1152 \text{ FT/MIN}$$

The **Velocity Correction Factor** is used to increase the bending and surface compressive stresses due to tooth impact action when the load is transferred from one tooth to another. Such a correction factor is necessary in order to take into account geometric imperfections in tooth profile and spacing errors generated by manufacturing equipment tolerances, heat treating distortion, and load distortion.

$$VCF = \sqrt{\frac{78}{78 + \sqrt{PLV}}} \quad (17)$$

$$VCF = \sqrt{\frac{78}{78 + \sqrt{1152}}} = .835$$

Other magnitude factors may be used. The magnitude of the factor is an arbitrary decision, but using one increases the reserve factors and improves the gear life.

The **Tangential Load** is the torque divided by the radius.

$$TL = 2T / NPD \quad (18)$$

$$TL = 2 \times 5000/2.2 = 4545 \text{ lbs.}$$

The **Base Circle Diameter** is the diameter of the circle from which the tooth involute profile is developed as shown in Figures 1 and 2.

$$BCD = NPD \times COS(NPA) \quad (19)$$

$$\text{Pinion } BCD = 2.200 \times COS(20) = 2.067 \text{ in.}$$

$$\text{Gear } BCD = 6.729 \times COS(20) = 6.323 \text{ in.}$$

The **Base Pitch** is the circular tooth size on the base circle diameter. It is obtained by dividing the base circle diameter by the number of teeth. It must be equal for any gears to mesh properly.

$$BP = BCD \times 3.14/TE \quad (20)$$

$$\text{Pinion } BP = \text{Gear } BCD = \frac{2.067 \times 3.14}{17} = .382$$

The **Profile Contact Ratio** must be greater than one so that constant angular velocity is maintained and the tooth load is properly being transferred. It is determined as follows:

$$PCR = \frac{\sqrt{POR^2 - PBR^2} + \sqrt{GOR^2 - GBR^2} - \sqrt{C^2 - (PBR + GBR)^2}}{BP} \quad (21)$$

$$PCR = \frac{\sqrt{1.225^2 - 1.033^2} + \sqrt{3.489^2 - 3.161} - \sqrt{4.464^2 - (1.033 + 3.161)^2}}{.382} = 1.582$$

The **Helical Contact Ratio** shows the number of teeth that carry the load at any given time. Helical gears have a smoother load transfer from tooth to tooth than spur gears where the load is being transferred nearly instantaneously. Helical gears should be designed with a contact ratio of greater than one to generate a continuous load transfer, they will also run quieter than gears with a helical contact ratio of higher than one and the ones with a contact ratio of less than one. But no difference in life durability can be observed between gears with a contact ratio of less than one or even spur gears that naturally have a helical contact ratio of zero. This may be because sound is produced by an insignificant amount of energy dissipation.

$$HCR = \frac{FW \times TAN(HA)}{CP} \quad (22)$$

$$HCR = \frac{2.00 \times TAN(15)}{.406} = 1.319$$

The **Surface Compressive Stress** occurs at the gear mesh on the teeth where the load is transferred from one gear tooth to another. It is calculated as follows:

$$SC = \sqrt{\frac{.35 \times TL}{FW \times SIN(NPA) \times VCF}\left(\frac{1}{PNPD} + \frac{1}{GNPD}\right)E} \quad (23)$$

$$SC = \sqrt{\frac{35 \times 4545}{2.0 \times SIN\ 20 \times 835}\left(\frac{1}{2.200} + \frac{1}{6.729}\right) \times 30 \times 10^6} = 224.488 \text{ PSI}$$

The width of the narrower gear should be used as it determines the amount of face contact.

Table 1 may be used as a guide for acceptable levels of surface compressive stress.

The **Tooth Bending Stress** may be calculated by using the AGMA strength factor (J). It can be calculated as follows:

$$SB = \frac{TL \times NDP}{VCF \times FW \times J} \quad (24)$$

$$\text{Pinion } SB = \frac{4545 \times 8}{.835 \times 2.25 \times .23} = 84.144 \text{ PSI}$$

$$\text{Gear } SB = \frac{4545 \times 8}{.835 \times 2.00 \times .28} = 77.759 \text{ PSI}$$

It is preferred to make the small diameter gear slightly wider to help reduce the bending stress because the strength factor grows in relation to the number of teeth and it always helps reduce the bending stress of the larger gear.

When balanced fatigue life is desired, the stress of the gear with the smaller number of teeth should be lower because it accumulates more cycles per tooth. The bending stress may be proportionally reduced on both gears by using a lower diametral pitch and reducing the number of teeth in each gear to maintain nearly the same center distance as long as the pinion does not have too low a number of teeth.

Designing a set of gears following the above equations and guidelines will produce a long life service if, in addition to that, the manufacturing accuracies are satisfactory and adequate lubrication is provided. It is important to understand that the oil in a gear box has two functions: 1) to provide a lubricating film that maintains metal separation, and 2) to carry the heat away from the gear teeth and into the atmosphere. The heavier the oil is, the thicker the separating film is and, consequently, longer gear and bearing life results from improved lubrication. Using a reserve factor is entirely arbitrary, but it is recommended that one should be used. Last, but not least, testing is a must in order to verify that the engineering assumption used and the design state were reasonably good.

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