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Spur and Helical Gear Design

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ABSTRACT

The purpose of this article is to provide the design formulas and acceptable stress levels so that calculations of the gear tooth geometry, surface compressive, and bending stresses at which the gears will operate in a known application can be made. It will be necessary to know the complete gear information for all gears, such as number of teeth, diametral pitch, pressure angle, gear width, type of material, and type of heat treatment, if any.

The surface compressive and bending stresses are based on American Gear Manufacturers Association (AGMA) formulas and tables, and the applications stress limits are based on Machine Design recommended safe stress levels and authors past experience. By combining the proper number of teeth with the proper size tooth, the proper pressure angle, the proper material and heat treatment, a durable gear box can be designed to offer a long life under any operating conditions.

IT IS THOUGHT that scientist Euler was the first one to recognize that the involute profile form of the gear tooth provides constant angular velocity. It was not until the beginning of this century that this tooth form became widely used and now it is almost exclusively used for transfer of heavy or light loads at low or high angular velocity.

A simple development of the involute profile is obtained by unwinding a wire off a cylinder and having a pencil tied to the end of the wire and drawing a line on a sheet of paper while unwinding the wire as illustrated on Figure 1. In this case the cylinder and the paper are stationary, and the pencil moves. Another way of describing an involute profile is by wrapping a belt in a figure "8" on two parallel cylinders, attaching a pencil on the belt and rotating the cylinder on a sheet of paper that is attached to one cylinder as shown on Figure 2. If two pencils are attached on the points A and B on the belt and cylinder L has a sheet of paper with diameter C attached to itself and is being rotated in both directions, the pencils will draw two curves from the paper outside diameter to the cylinder diameter D. These curves have an involute profile and can be the profile of a gear tooth. If the belt had zero thickness so that the drawing pencil can travel on diameter D or E, then the D and E diameters become the gear base circle diameters of the respective cylinders L and R.

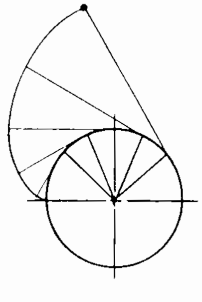

FIGURE 1

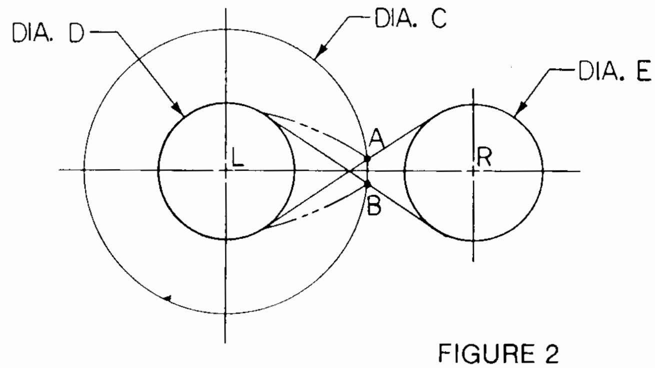

FIGURE 2

THE INVOLUTE:

Since this paper provides the equations of involute profile gears, it is appropriate to discuss the involute function. The involute angle is formed by a straight line that passes through the gear center and crosses at any point on the gear tooth profile as shown on Figure 3. The numerical value of the involute angle is found by subtracting the angle expressed in radius from the tangent of the same angle as shown by equation 1.

$$INV(PA) = TAN(PA) - PA \tag{1}$$

From the above equation it can be seen that it is easy to solve for the involute numerical value when the angle is known, but the solution for the angle when

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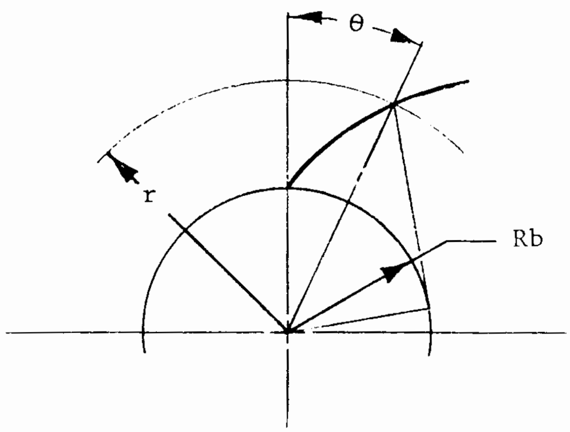

FIGURE 3

Table 1

Material	Type of Heat Treatment	Hardness	Tensil Strength (PSI)	Allowable Surface Compressive Stress	Allowable Bending Stress
Low Carbon Steel	Carborize & Case Harden	Rc 60	325,000	200,000	50,000
Low Carbon Steel	Carborize & Case Harden	Rc 55	285,000	180,000	45,000
Low Carbon Steel	Carborize & Case Harden	Rc 50	245,000	155,000	40,000
Low Carbon Steel	Carborize & Case Harden	Rc 45	212,000	132,000	33,000
Low Carbon Steel	Carborize & Case Harden	Rc 40	186,000	115,000	28,000
Med. Carb. Steel	Full Hardened	BHN 440	223,000	140,000	30,000
Med. Carb. Steel	Full Hardened	BHN 360	180,000	110,000	26,000
Med. Carb. Steel	Full Hardened	BHN 300	150,000	90,000	22,000
Med. Carb. Steel	Full Hardened	BHN 240	116,000	70,000	17,000
Med. Carb. Steel	Full Hardened	BHN 180	80,000	50,000	12,000

the numerical value is given must be made by trial and error because the angle expressed in radius and its tangent must be found simultaneously. When using a computer to find the angle it may be desired to generate a table from ten to forty degrees in steps of .00001 and match a given involute numerical value to its respective angle in the table. This approach should yield sufficient accuracy as the variation will be less than .0001 inch in most cases. Some ingenuity in programming the computer for this trial and error reiteration may reduce the calculating time substantially.

DESIGN PROCEDURE:

To better illustrate how to design a set of gears an example of two helical gears will be calculated step by step with the writing of the basic formulas. The pinion will have 17 teeth and the gear will have 52 teeth. This pair of gears provides a reduction ratio of 52 divided by 17 = 3.059. The equations can be used interchangeably between spur and helical gears since the helical angle of a spur gear is zero and the cosine of zero degrees is one. Even though the calculating of one gear could illustrate the utilization of the gear design equations, two gears were selected for calculations to show how to modify the addendums to eliminate undercut on the pinion and because contact ratio and operating stresses require the use of two gears to achieve power transfer.

Once the gear reduction ratio was made, one can proceed to select the diametral pitch, normal pressure angle, and the helical angle. This is a trial and error selection based on availability of parts or tooling and the hope that this first selection is suitable for the required performance. If the selection is not suitable, the calculated results may be compared with the allowable stresses shown on Table 1 for the respective materials and a second set of calculations may be initiated making the gears larger or smaller as desired to obtain the recommended operating stress levels.

Before the equations are written it is necessary to establish the symbols of the variables to be used. The following is a list of definitions of the symbols. Some of these definitions are illustrated in Figure 4.

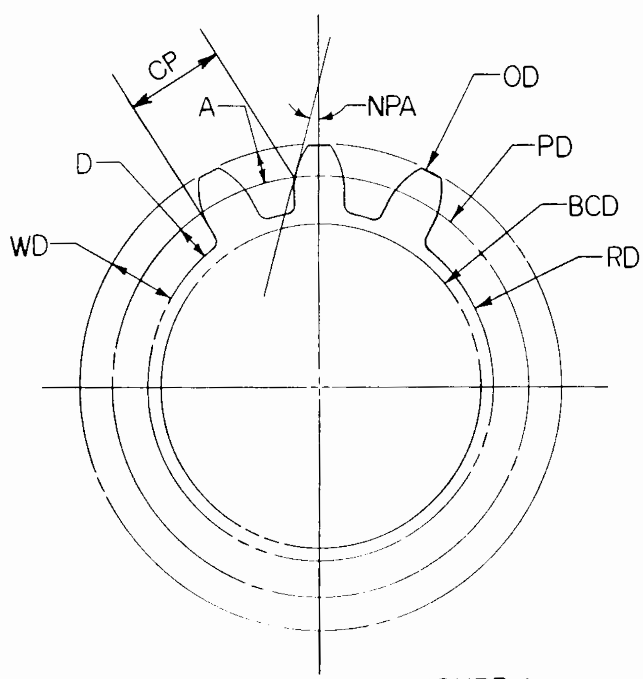

FIGURE 4

A . . . . Addendum. The length of tooth from the nominal pitch diameter to the outside diameter.

AD . . . . . Diameter at any point on the tooth.

APA . . . . Pressure angle in the plane of rotation at any given diameter.

ATTT . . . Transverse tooth thickness at any given diameter.

BCD . . . . Base circle diameter.

BL ...... Backlash is the operating clearance between the teeth.
BP ...... Base pitch.
CL ...... Clearance between the tooth tip and root diameter of the mating gear.
CP ...... Circular pitch.
D .... Dedendum. The length of tooth from the nominal pitch diameter to the root diameter.
DR ...... Change in pinion radius to eliminate undercut.
DTTT .... Change in transverse tooth thickness resulting from change in pinion addendum to eliminate undercut.
E .... Modulus of elasticity (30×10^6 PSI for steel)
FW ..... Gear face width.
G .... Gear. The gear is the driven gear.
GBR .... Gear base radius.
GMTTT .. Gear modified transverse tooth thickness at the nominal pitch diameter.
GNPD ... Gear nominal pitch diameter.
GOR .... Gear outside radius.
GPA .... Generating pressure angle. It is always normal to the tooth surface and it equals the angle at the nominal pitch diameter.
GTTT .... Gear transverse tooth thickness.
GWPD ... Gear working pitch diameter.
HA ...... Helix angle at the nominal pitch diameter. For spur gears use a helical angle of zero degrees.
HCR .... Helical contact ratio.
HP ...... Horsepower.
L .... Lead. A point anywhere on the tooth of a helical gear that is found again when the tooth makes a 360 degree spiral on the gear.
NDP .... Normal diametral pitch. This indicates how many gear teeth are on a spur gear that has a one inch nominal pitch diameter.
NPA ..... Normal pressure angle. This is the angle that a tangent line on the tooth at the pitch diameter forms with a line that joins the center tooth thickness with the gear center.
NPD .... Nominal pitch diameter.
NTT ..... Normal tooth thickness at the nominal pitch diameter.
NWPA ... Normal working pressure angle.
OD ..... Outside diameter.
P .... Pinion. The pinion is the driving gear.
PA ...... Pressure angle in the plane of rotation at any point on the tooth profile.
PBR ..... Pinion base radius.
PCR ..... Profile contact ratio.
PMTTT .. Pinion modified transverse tooth thickness at the nominal pitch diameter.
PNPD ... Pinion nominal pitch diameter.
POR ..... Pinion outside radius.
PWPD ... Pinion working pitch diameter.
R .... Corner radius between hob outside diameter and the pressure angle face. See Figure 6.
RD ..... Root diameter.
RPA ..... Pressure angle in the plane of rotation at the nominal pitch diameter.
RPM .... Gear speed expressed in revolutions per minutes.
RU ...... Length of radial undercut.
SB ...... Bending stress.
SC ...... Surface compressive stress.
T .... Torque.
TE ...... Number of teeth.
TL ...... Tangential load.
TTT ..... Transverse tooth thickness at the nominal pitch diameter.
VCF ..... Velocity correction factor.
WD ..... Whole depth. The radial length of the tooth from the root diameter to the outside diameter.
WPD .... Working pitch diameter.

It is most desirable to first establish the torque and reduction ratio at which the gears have to operate so that when calculating a matched set of gears the stress levels are established.

To determine if this set of gears is adequate, the calculated stress level has to be compared with the permissible stress levels shown on Table 1. If the gears have stress levels that are too high, larger gears must be selected so that adequate life is attained. The first approach of selecting gears with increased load carrying capacity should be made by increasing the gear width because increase in the cost of material is proportional to the width. When it is necessary to increase the gear load carrying capacity by increasing the diameters it should be taken into account that the pitch line velocity increases and the cost of material increases with the square of the diameter increase.

The **Nominal Pitch Diameter** is an invisible circular line slightly above the mid-height of the tooth on unmodified gears. At this point pure rolling exists when the gears rotate only when the working pitch diameter coincides with the nominal pitch diameter. Surface sliding is generated and increases as the load lines are farther and farther away from the working pitch diameter either towards the outside diameter or the root diameter.

The gear data at the nominal pitch diameter is the basic gear information, therefore it must be unchanged in order to attain proper gear mesh. In addition to that, the profile contact ratio must be inspected and maintained substantially above one so that the load is transfered smoothly from one gear to another.

$$NPD = \frac{TE}{NDP \times COS(HA)} \quad (2)$$

The **Working Pitch Diameter**[1] is at a different place than the nominal pitch diameter only on gears that operate on a modified center distance. No surface sliding occurs at this point, however, these gears will have sliding at the nominal pitch diameter. The dimensions over balls, bending stress and all other gear calculations should be based on tight mesh tooth thicknesses at zero backslash at the working pitch diameter and adjusted for the desired amount of backslash.

$$WPD = NPD \frac{COS\ (GPA)}{COS\ (NWPA)} \quad (3)$$

This is a very useful tool because it enables changing the number of teeth on a gear by one or two teeth and still allow the use of an existing center distance. Such a requirement may occur when it is desired to change a gear ratio to a lesser variation than by adding one tooth to the pinion and at the same time subtracting one tooth from the gear in order to maintain a given center distance.

This approach of having a working pitch diameter different than the nominal pitch diameter can also be used to determine a center distance for any two nonstandard gears. One can use this equation by trial and error until the transverse tooth thickness at the nominal pitch diameter of both gears matches that of the given gear data. Use of a computer should make this trial and error calculation a non-time consuming process.

This approach is necessary when desiring to eliminate undercut from a gear with fewer teeth than are shown on Table 2 and using it with a gear of standard design.

When following this procedure of removing undercut the tooth thickness is being increased in addition to increasing the outside diameter. It should be obvious that a larger center distance is required whenever one gear is being replaced by another gear which has a larger tooth thickness at the nominal pitch diameter.

If the center distance cannot be increased when a gear with an increased transverse tooth thickness is replacing a standard gear, then the matching gear tooth thickness must be reduced by an equivalent amount so that this pair of gears can be assembled. It must be remembered to provide operating clearance between the teeth so that they will never operate at tight mesh.

The **Center Distance** as expressed by equation (4) is determined by the nominal pitch diameters of the two meshing gears that have an unmodified tooth thickness as expressed by equation (15) or only if the modification is such that the increase in tooth thickness at the nominal pitch diameter is equal to the decrease in the tooth thickness of the mating gear.

$$CD = \frac{PNDP + GNPD}{2} \quad (4)$$

The center distance for gears where one or both gears have the transverse tooth thickness that is not equal to that one expressed by equation (15) and if the sum of both does not equal twice the quantity of equation (15) may be found by the following equation:

$$CD = \frac{PNPD + GNPD}{2} \frac{COS\ (GPA)}{COS\ (NWPA)} = \frac{PWPD + GWPD}{2} \quad (5)$$

The reader who is not familiar with this approach should read the section of "Working Pitch Diameter" to understand its full benefit. It is worth emphasizing again that when calculating a center distance based on a working pitch diameter all gear dimensions at the nominal pitch diameter must be maintained. The transverse tooth thickness at the working pitch diameter at zero backlash will result in tight mesh, therefore, some operating clearance must be provided by thinning either tooth or both.

The **Circular Pitch** is defined as the circular sum of one tooth thickness and one space width at the working pitch diameter for modified gears, or at the nominal pitch diameter for unmodified gears.

$$CP = \frac{\pi \times WPD}{TE} = \frac{\pi}{NDPX\ COS\ (HA)} \frac{COS\ (GPA)}{COS\ (NWPA)} \quad (6)$$

The above is the general equation and when the gears are standard the generating pressure angle (GPA) is equal to the normal working pressure angle (NWPA) and equation (6) can be rewritten as follows:

$$CP = \frac{\pi}{NDPX\ COS\ (HA)} \quad (7)$$

For spur gears the above equation can be further simplified as follows:

$$CP = \frac{\pi}{NDP} \quad (8)$$

Equation (8) makes the definition of the Nominal Diameter Pitch become obvious as the number of spur teeth that can be made on a one inch pitch diameter gear. This is also the equation of the pitch of a rack or a hob. For a hob the pitch is in the plane that is perpendicular to the hob spiral angle.

The **Addendum** is the distance from the nominal pitch diameter to the outside diameter. Its length is customarily the reciprocal of the normal diametral pitch. It can be any length so long as it yields a contact ratio greater than one under the worst tolerance accumulation conditions and the tip thickness is adequate.

$$\text{Addendum:}\quad A = \frac{1}{NDP} \quad (9)$$

The **Dedendum** is the distance from the nominal pitch diameter to the root diameter. It must be greater than the addendum to provide clearance between the root diameter and the outside diameter of the mating gear. It is customarily 1.157 times the length of the addendum.

$$\text{Dedendum:}\quad D = \frac{1.157}{NDP} \tag{10}$$

Clearance is the space between the tip of the tooth and the root diameter of the mating gear.

$$CL = \frac{.157}{NDP} \tag{11}$$

It should be noted that this clearance is for gears without backlash. Since all gears must have backlash, it follows that the above clearance becomes larger by an amount that is equal to the cutter radial infeed that generates the backlash.

The **Root Diameter** is the sum of the nominal pitch diameter minus two dedendums. In reality, the root diameter is slightly smaller because the gear teeth are slightly thinned to provide operating clearance. The reduction in root diameter corresponds to the amount of backlash provided by each gear. The dimensional reduction in the root diameter can be calculated but it has no significant effect on the gear bending strength and no effect at all on the surface compressive stress.

$$\text{Root Diameter} = NPD - 2D \tag{12}$$

The **Lead** is a point anywhere on the tooth that is found again in the axial plane when that tooth makes a 360 degree spiral on the gear. A change in the manufactured lead from the theoretical lead indicates a change of the helix angle that may result in uneven load distribution across the tooth face. When the lead error per tooth is equal and in the same direction for both the pinion and the gear, uniform load distribution is achieved even though a lead error exists.

Because spur gears have no helical angle the lead is infinite. A lead inspection of a spur gear only shows how well the tooth was manufactured in the axial plane and indicates how good a load distribution will be achieved.

$$\text{Lead:}\quad L = \frac{NPD \times 3.14}{TAN(HA)} \tag{13}$$

The **Normal Tooth Thickness** at the nominal pitch diameter for an unmodified gear is:

$$NTT = \frac{3.14}{2 \times NDP} \tag{14}$$

The tooth should be thinned by several thousandths of one inch to provide operating backlash. Note that for a spur gear the normal tooth thickness at the nominal pitch diameter is equal to the transverse tooth thickness at the nominal pitch diameter.

The **Transverse Tooth Thickness** is measured at the nominal pitch diameter and it is in the plane of rotation. It should be noted that even though the normal tooth thickness remains the same for any helical angle, the transverse tooth thickness increases as the helical angle increases. This dimension is used in calculating the dimension over rolls for manufacturing. This tooth thickness should be slightly reduced so that operating clearance on the non-loaded face of the tooth is provided. If operating clearance does not exist, radial loads can be developed that are many times higher than the designed loads and premature failures will result.

The transverse tooth thickness for a helical gear set where both gears have equal tooth thickness at the nominal pitch diameter can be calculated as follows:

$$TTT = \frac{NPD \times 3.14}{2 \times TE} \tag{15}$$

The tooth thickness should be made nearly equal for both gears so that the tip thickness does not become too small.

When it is desirable to eliminate undercut without changing the center distance it is possible to accomplish that by retracting the hob until the root diameter is equal or larger than the base circle diameter. To maintain a good tooth proportion it is necessary to increase the outside diameter by the same amount of increase in the root diameter. This is referred to be an increased addendum gear. The outside diameter of the mating gear must be reduced by the same amount that the previous gear was increased. This is referred to as a short addendum. Equation (22) provides the amount of increased transverse tooth thickness for a given change in radial hob position which is equal to the amount of undercut as well as the increase in addendum. The transverse tooth thickness for a gear with a modified addendum and no undercut becomes:

$$PMTTT = TTT + DTTT \tag{16}$$

This is the equation for a gear with an increased tooth thickness at the nominal pitch diameter. The transverse tooth thickness of the mating gear may be found by simply subtracting the above quantity from the circular pitch. The equation becomes:

$$GMTTT = CP - PMTTT \tag{17}$$

The transverse tooth thickness at any point on the tooth as shown on Figure 5 may be found by the following equation:[2]

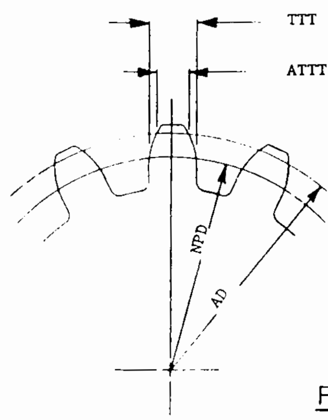

FIGURE 5

$$ATTT = AD\left[\frac{TTT}{NPD} + INV(RPA) - INV(APA)\right] \quad (18)$$

It is emphasized that all of the above quantities are in the plane of rotation. The involute at a desired point on the tooth, INV (APA), may be found by first solving for the transverse pressure angle at that point by using equation (19) and then the involute is found by using equation (1).

This method of finding the tooth thickness at any desired point on the tooth is useful in finding the tip thickness for modified gears to prevent the possibility that they may become too thin or pointed. It can also be used to find the tooth thickness at the nominal pitch diameter when a given tooth thickness must be maintained at the working pitch diameter.

The three examples shown at the end should illustrate the calculations discussed in this paper. The first example of calculations is of an unmodified pair of gears the pinion has undercut and consequently the teeth have a lower bending strength. On the second example of calculations the undercut is removed by increasing the pinion addendum and decreasing the gear addendum. The center distance remains standard as in the first example. Because the gear outside diameter and the tooth thickness have been reduced, it is a special gear designed for a particular application. The third example shows a set of gears where the pinion undercut has been removed by increasing the addendum as was done on the second example, but the gear is not modified as illustrated in the first example. Because the pinion has a larger transverse tooth thickness and the gear has a standard tooth thickness the center distance is larger than in the first and second example to accommodate the added tooth thickness.

The **Pressure Angle in the plane of rotation at the nominal pitch diameter** is the same as the normal pressure angle for spur gears, but it is larger than the normal pressure angle for helical gears. It is calculated as follows:

$$RPA = ARCTAN\ (TAN(NPA)/COS(HA)) \quad (19)$$

Equation (20) shows that the product of the diameter and the pressure angle in the plane of rotation at that diameter equals the product of a diameter at any place on the tooth times the pressure angle at that place.

$$NPD \times COS\ (RPA) = AD \times COS\ (APA) \quad (20)$$

$$COS\ (APA) = \frac{NDP}{AD} COS\ (RPA) \quad (21)$$

The **Transverse Pressure Angle** at any point on the tooth profile as shown by equation (21). For spur gears it is the same as the nominal pressure angle, however, for helical gears the transverse pressure angle and the nominal pressure angle are different.

Undercut may be avoided by making the root diameter equal or larger than the base diameter.

Undercut reduces the bending strength of the tooth leading to premature fatigue failure, and it also may reduce the profile contact ratio if the undercut extends into the active profile.

Generating Pressure Angle	Spur Gear	Helical Gear Angle			
		10	20	30	45
14.5°	32	31	27	21	12
20°	18	17	15	12	7
25°	12	12	10	8	5

Table 2
Minimum number of teeth without undercut.

Whenever it is desired to eliminate the undercut it can be done by increasing the pinion addendum by the amount of radial undercut and reducing the gear addendum by an equal amount. It is also necessary to modify the tooth thickness so that the pinion does not become too weak and to avoid the possibility that the tip may become pointed. The pinion transverse tooth thickness may be increased by the following formula (also see Figure 6):

$$DTTT = \frac{DR \times TAN(NPA) \times 2}{COS(HA)} \quad (22)$$

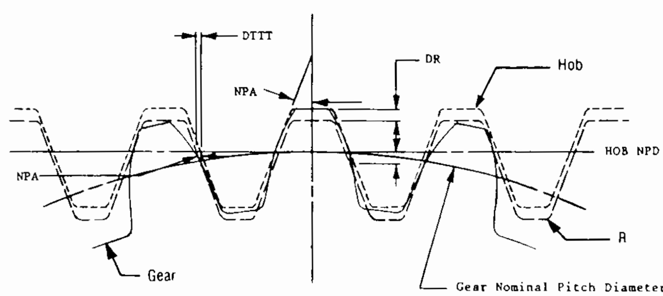

FIGURE 6

The **Pitch Line Velocity** is a function of pitch diameter and speed. It is calculated in feet per minute as follows:

$$PLV = NPD \times 3.14 \times RPM/12 \quad (23)$$

The **Velocity Correction Factor** is used to increase the calculated bending and surface compressive stresses due to tooth impact action whe, the load is transferred from one tooth to another and due to concentrated loads crested by non uniform load distribution. Such a correction factor is necessary in order to take into account geometric imperfections in tooth profile and spacing errors generated by manufacturing equipment tolerances, heat treating distortion, and load distortion.

$$VCF = \sqrt{\frac{78}{78 + \sqrt{PLV}}} \quad (24)$$

Other magnitude factors may be used. The magnitude of the factor is an arbitrary decision, but using one increases the reserve factors and improves the gear life.

The **Tangential Load** is the torque divided by the radius.

$$TL = 2T \ / \ NPD \tag{25}$$

The **Base Circle Diameter** is the diameter of the circle from which the tooth involute profile is developed as shown in Figures 1 and 2.

$$BCD = NPD \times COS(NPA) \tag{26}$$

The **Base Pitch** is the circular tooth size on the base circle diameter. It is obtained by dividing the base circle diameter by the number of teeth. It must be equal for any gears to mesh properly.

$$BP = BCD \times 3.14/TE \tag{27}$$

The **Profile Contact Ratio** must be greater than one so that constant angular velocity is maintained and the tooth load is properly being transferred. It is determined as follows:

$$PCR = \frac{\sqrt{POR^2 - PBR^2} + \sqrt{GOR^2 - GBR^2} - \sqrt{C^2 - (PBR + GBR)^2}}{BP} \tag{28}$$

The **Helical Contact Ratio** shows the number of teeth that carry the load at any given time. Helical gears have a smoother load transfer from tooth to tooth than spur gears where the load is being transferred nearly instantaneously. Helical gears should be designed with a contact ratio greater than one to generate a continuous load transfer. They will also run quieter than gears with a helical contact ratio of less than one. But no difference in life durability can be observed between gears with a contact ratio of less than one or even spur gears that naturally have a helical contact ratio of zero. This may be because sound is produced by an insignificant amount of energy dissipation.

$$HCR = \frac{FW \times TAN(HA)}{CP} \tag{29}$$

The **Surface Compressive Stress** occurs at the gear mesh on the teeth where the load is transferred from one gear tooth to another. It is calculated as follows:

$$SC = \sqrt{\frac{.35 \times TL}{FW \times SIN(NPA) \times VCF}\left(\frac{1}{PNPD} + \frac{1}{GNPD}\right)E} \tag{30}$$

The width of the narrower gear should be used as it determines the amount of face contact.

Table 1 may be used as a guide for acceptable levels of surface compressive stress.

The **Tooth Bending Stress** may be calculated by using the AGMA[3] strength factor (J). It can be calculated as follows:

$$SB = \frac{TL \times NDP}{VCF \times FW \times J} \tag{31}$$

It is preferred to make the small diameter gear slightly wider to help reduce the bending stress because the strength factor grows in relation to the number of teeth and it always helps reduce the bending stress of the larger gear.

When balanced fatigue life is desired, the stress of the gear with the smaller number of teeth should be lower because it accumulates more cycles per tooth. The bending stress may be proportionally reduced on both gears by using a lower diametral pitch and reducing the number of teeth in each gear to maintain nearly the same center distance as long as the pinion does not have too low a number of teeth.

Designing a set of gears following the above equations and guidelines will produce a long service life if, in addition to that, the manufacturing accuracies are satisfactory and adequate lubrication is provided. It is important to understand that the oil in a gear box has two functions: 1) to provide a lubricating film that maintains metal separation, and 2) to carry the heat away from the gear teeth and into the atmosphere. The heavier the oil is, the thicker the separating film is and, consequently, longer gear and bearing life results from improved lubrication. Using a reserve factor is entirely arbitrary, but it is recommended that one should be used. Last, but not least, testing is a must in order to verify that the engineering assumption used and the design state were reasonably good.

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IN THIS EXAMPLE THE PINION AND THE GEAR HAVE STANDARD DESIGN. THE PINION HAS UNDERCUT AND THEY OPERATE ON A STANDARD CENTER DISTANCE.

POWER ENGINEERING & MANUFACTURING LTD.

724 SYCAMORE ST. WATERLOO,IOWA 50703 319-232-2311

HELICAL GEAR DESIGN 8-2

GEAR BOX NUMBER	SAE PAPER EXAMPLE	
PART NUMBER	P-C-31	G-C-24
PART NAME	PINION	GEAR
NUMBER OF TEETH	17.0000	52.0000
NORMAL DIAMETRAL PITCH	8.0000	
NORMAL PRESSURE ANGLE	20.0000	
HELIX ANGLE	30.0000	
DIRECTION OF HELIX	LH	RH
NORMAL WORKING PRESSURE ANGLE	20	
FACE WIDTH	1.7500	1.5000
NOMINAL PITCH DIAMETER	2.4537	7.5056
WORKING PITCH DIAMETER	2.4537	7.5056
TTT AT WPD AT 0 BL	0.2267	0.2267
TTT AT NPD AT 0 BL	0.2267	0.2267
OUTSIDE DIAMETER	2.7037	7.7556
ROOT DIAMETER	2.1737	7.2256
WHOLE DEPTH	0.2650	0.2650
PIN DIAMETER	0.2500	0.2500
DIMENSION OVER PINS	2.8479	7.9245
TOLERANCE +/-	0.0032	0.0037
MIN/ADD BACKLASH	0.0030	0.0030
LEAD	13.3518	40.8407
HOB RETRACTION DISTANCE	0.0000	0.0000
BASE DIAMETER	2.2621	6.9193
NORMAL TOOTH TIP THICKNESS	0.0908	0.0999
STANDARD OD TO RD CLEARANCE	0.0200	
OD TO RD CLEARANCE	0.0150	0.0150
TORQUE	1500.0000	4588.2353
HORSEPOWER	42.8571	
RATIO	3.0588	
RPM	1800.0000	588.4615
PITCH LINE VELOCITY	1156.2971	
AXIAL LOAD	751.1843	
PROFILE CONTACT RATIO	1.3460	
HELICAL CONTACT RATIO	1.9099	
CENTER DISTANCE	4.9796	
SURFACE COMPRESSIVE STRESS	131362.2439	
BENDING STRESS	0.0000	13197.8080
ADDENDUM	0.1250	0.1250
CUTTER ADDENDUM/DEDENDUM	0.1400	0.1250

DATE 07/05/84

IN THIS EXAMPLE THE PINION HAS INCREASED ADDENDUM TO ELIMINATE UNDERCUT. THE GEAR IS OF STANDARD DESIGN AND THEY BOTH OPERATE ON INCREASED CENTER DISTANCE.

POWER ENGINEERING & MANUFACTURING LTD.

724 SYCAMORE ST. WATERLOO,IOWA 50703 319-232-2311

HELICAL GEAR DESIGN 8-2

GEAR BOX NUMBER	SAE PAPER EXAMPLE	
PART NUMBER	P-C-32	G-C-24
PART NAME	PINION	GEAR
NUMBER OF TEETH	17.0000	52.0000
NORMAL DIAMETRAL PITCH	8.0000	
NORMAL PRESSURE ANGLE	20.0000	
HELIX ANGLE	30.0000	
DIRECTION OF HELIX	LH	RH
NORMAL WORKING PRESSURE ANGLE	21.111	
FACE WIDTH	1.7500	1.5000
NOMINAL PITCH DIAMETER	2.4537	7.5056
WORKING PITCH DIAMETER	2.4777	7.5789
TTT AT WPD AT 0 BL	0.2609	0.1969
TTT AT NPD AT 0 BL	0.2688	0.2267
OUTSIDE DIAMETER	2.8037	7.7556
ROOT DIAMETER	2.2737	7.2254
WHOLE DEPTH	0.2650	0.2651
PIN DIAMETER	0.2500	0.2500
DIMENSION OVER PINS	2.9223	7.9244
TOLERANCE +/-	0.0030	0.0037
MIN/ADD BACKLASH	0.0030	0.0030
LEAD	13.3518	40.8407
HOB RETRACTION DISTANCE	0.0500	0.0000
BASE DIAMETER	2.2621	6.9193
NORMAL TOOTH TIP THICKNESS	0.0742	0.0999
STANDARD OD TO RD CLEARANCE	0.0200	
OD TO RD CLEARANCE	0.0137	0.0136
TORQUE	1500.0000	4588.2353
HORSEPOWER	42.8571	
RATIO	3.0588	
RPM	1800.0000	588.4615
PITCH LINE VELOCITY	1167.5900	
AXIAL LOAD	756.6665	
PROFILE CONTACT RATIO	1.2635	
HELICAL CONTACT RATIO	1.9099	
CENTER DISTANCE	5.0283	
SURFACE COMPRESSIVE STRESS	127279.2415	
BENDING STRESS	11556.0302	13202.1456
ADDENDUM	0.1750	0.1250
CUTTER ADDENDUM/DEDENDUM	0.1400	0.1250

DATE 07/05/84

IN THIS EXAMPLE THE PINION HAS AN INCREASED ADDENDUM TO ELIMINATE UNDERCUT. THE GEAR HAS A REDUCED ADDENDUM TO ALLOW THE GEARS TO OPERATE ON STANDARD CENTER DISTANCE.

POWER ENGINEERING & MANUFACTURING LTD.

724 SYCAMORE ST. WATERLOO,IOWA 50703 319-232-2311

HELICAL GEAR DESIGN 8-2

GEAR BOX NUMBER	SAE PAPER EXAMPLE	
PART NUMBER	P-C-31	G-C-25
PART NAME	PINION	GEAR
NUMBER OF TEETH	17.0000	52.0000
NORMAL DIAMETRAL PITCH	8.0000	
NORMAL PRESSURE ANGLE	20.0000	
HELIX ANGLE	30.0000	
DIRECTION OF HELIX	LH	RH
NORMAL WORKING PRESSURE ANGLE	20	
FACE WIDTH	1.7500	1.5000
NOMINAL PITCH DIAMETER	2.4537	7.5056
WORKING PITCH DIAMETER	2.4537	7.5056
TTT AT WPD AT 0 BL	0.2688	0.1847
TTT AT NPD AT 0 BL	0.2688	0.1847
OUTSIDE DIAMETER	2.8037	7.6556
ROOT DIAMETER	2.2737	7.1256
WHOLE DEPTH	0.2650	0.2650
PIN DIAMETER	0.2500	0.2500
DIMENSION OVER PINS	2.9223	7.8325
TOLERANCE +/-	0.0030	0.0039
MIN/ADD BACKLASH	0.0030	0.0030
LEAD	13.3518	40.8407
HOB RETRACTION DISTANCE	0.0500	-0.0500
BASE DIAMETER	2.2621	6.9193
NORMAL TOOTH TIP THICKNESS	0.0742	0.1040
STANDARD OD TO RD CLEARANCE	0.0200	
OD TO RD CLEARANCE	0.0150	0.0150
TORQUE	1500.0000	4588.2353
HORSEPOWER	42.8571	
RATIO	3.0588	
RPM	1800.0000	588.4615
PITCH LINE VELOCITY	1156.2971	
AXIAL LOAD	751.1843	
PROFILE CONTACT RATIO	1.2841	
HELICAL CONTACT RATIO	1.9099	
CENTER DISTANCE	4.9796	
SURFACE COMPRESSIVE STRESS	131362.2439	
BENDING STRESS	11556.0302	14930.3905
ADDENDUM	0.1750	0.0750
CUTTER ADDENDUM/DEDENDUM	0.1400	0.1250
DATE 07/05/84		